INITIAL ELASTIC AND FRICTIONAL BEHAVIOR OF METAL INTERFACES

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Cambridge 39, Massachusetts

May 23, 1955

Secretary of the Faculty
Massachusetts Institute of Technology
Cambridge 39, Massachusetts

Dear Sir:

In accordance with the regulations of the Faculty, I submit herewith a thesis entitled "Initial Elastic and Frictional Behavior of Metal Interfaces," in partial fulfillment of the requirements for the degree of Master of Science (without specification).

Respectfully yours,

Leslie J. Williamson
Lieutenant, U.S. Coast Guard

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BEHAVIOR OF METAL INTERFACES

by

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B.S., U.S. Coast Guard Academy
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Department of Naval Architecture and
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Thesis Supervisor

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Chairman, Departmental Committee on Graduate Students

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Submitted to the Department of Naval Architecture and Marine Engineering on May 23, 1955, in partial fulfillment of the requirements for the degree of Master of Science

ABSTRACT

The object of this thesis is to investigate the reported existence of abnormal elastic effects in metal interfaces. In conjunction with this work, the initial frictional behavior at the metal interfaces was observed.

The method of investigation selected employed hollow cylindrical specimens composed of two mating parts placed end on end. The experimental apparatus utilized a combination of optical and mechanical means of measuring small angles of twist in the specimen. Various metals were tested under different conditions of normal load and surface finish.

Excellent conformity between observed values of twist and those predicted by elastic theory was achieved. The experimental results did not show any indication of excessive elastic angles of twist.

Investigation of the initial frictional behavior of the metal interfaces indicated that the value of the friction coefficient increased with incremental changes in the observed slip until the range of normally expected values was attained. In this range the curve flattened out and free sliding resulted.

In view of the results of this investigation, it is believed that the reported abnormal elastic conditions were the result of inaccuracies in the experimental method, and that further investigation along this line is not warranted.

Thesis Supervisor: Brandon G. Rightmire

Title: Associate Professor

ACKNOWLEDGMENT

To Professor Brandon G. Rightmire, who kindly consented to supervise this thesis, I wish to express my sincere appreciation of his helpful suggestions and encouragement during the course of the research.

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Finally, to Bertha Hornby, who typed the thesis, go my thanks for her careful work.

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I. INTRODUCTION

In a thesis,[1] Coyle and Stromberg reported the existence of abnormal elastic effects in metal interfaces. As a result of their work, they concluded that the asperities in steel interfaces contribute materially to the elastic twist. The elastic twist due to an interface appeared to decrease with increase in normal stress for a given value of maximum tangential stress. They also found that the effect of surface finish was affected by normal stress, in that a transition range existed for normal stress. This transition range separated the regions where elastic twist increased or decreased with the degree of surface finish. This appears to be a virgin investigation of elastic effects in metal interfaces, as a survey of the literature failed to disclose any previous work along this line.

Tomlinson, Thorpe, and Gough,[2] in a paper on fretting corrosion, reported that surfaces in contact under normal and tangential stresses have a comparatively high degree of tangential elasticity. "The surfaces appeared to yield under tangential stress in an elastic manner by an amount which may be many times greater than the smallest slip it is hoped to detect." This made their problem of measuring slip extremely difficult. In the second phase of this investigation, this problem was encountered where the displacements measured were of the same order of magnitude as the depth of the asperities

in the metal interfaces. The first objective of this investigation was the development of a test apparatus and an experimental procedure of sufficient sensitivity and accuracy so that the existing discrepancies between calculated theoretical and observed values of elastic twist would be eliminated or rationalized. Various methods of measuring elastic twist in a specimen were considered, as discussed in Appendix A.

The second phase of this work was a by-product of the original investigation. After extensive examination of the elastic effects at metal interfaces under various conditions of load and surface finish, it was decided that a study of the frictional behavior would be both interesting and valuable.

II. PROCEDURE

The test apparatus used in this work is illustrated in Figures I, II, and III.

The tubular specimens tested were machined from the following materials: (1) AISI C-1018, cold-finished, open-hearth, low-carbon steel; (2) AISI A-4140, heat-treated, stress-relieved, medium-carbon alloy steel; (3) hard-drawn, electrolytic, tough pitch copper rod; (4) 2S Aluminum.

The specimens were reamed out to an inside diameter of 0.191"; then turned down to 0.236" outside diameter. The test specimens were cut into two halves, each one inch long. A single two-inch specimen was made for each material, and used as a control specimen. The observed deflections of the control specimen (ψ_o) were compared with the computed deflections (ψ_c) as predicted by elastic theory, thus providing a check of the accuracy of the test runs. The effect of the interface could be determined by a comparison of control-specimen runs with test-specimen results.

The upper and lower halves of the test specimen are mounted as shown in Figure III. The contact surfaces were lapped to the desired finish with emery polishing paper of varying degrees of roughness. The specimen was clamped in a vee-type block during the polishing process, to insure that the test surfaces were ground perpendicular to the specimen axis. The specimens were carefully cleaned both before and after the

polishing process. (Reagent acetone was used as cleaning solvent.)

Two double-ended indicator arms were used, one on either side of the interface. Each arm was constructed with two 12" lengths of Type 321 stainless-steel tubing having 1/8" outside diameter and 0.005" wall thickness. The indicator arms were fastened to the specimen by a collar, as shown in Figure III. The cone-pointed set screws were used to obtain a knife-edge line from which the twist was transmitted. The pin-point indentations produced on the specimen by the set screws permitted length "L" to be accurately picked off the specimen with draftmen's dividers.

The alignment jig shown in Figure III was used to insure that the face of the indicator-arm collar, hence the plane of the set screws, was perpendicular to the axis of the specimen.

The lower half of the specimen rests on a machined and polished steel plate. For initial tests on the steel specimens a machined steel block was used, as shown in Figure I (1). The remainder of the specimens were mounted on the machined steel block shown in Figure III. This block has a securing collar which permits clamping of the bottom end of the specimen. After a final cleaning of the two contact surfaces, the upper half of the specimen was placed on the lower portion, and the weight rod then run up through the annulus of the specimen. The weight release (screw jack) permits the weight pan support rod to be run up and down as desired.

The threaded upper end of the rod was then screwed into a circular disc, separated from the top of the specimen by a ball thrust bearing as illustrated in Figure III. This bearing was used to isolate the specimen from any torsional vibration or movement of the normal load weight pan.

The sight edges are pictured in Figure I (2) and Figure III. The two sight edges were polished with 4/0 paper so that a clean, sharp sighting surface was obtained. Alignment of the two sight edges could be obtained by loosening either one or both of the clamp screws, and moving the edges into position as desired. The upper arm sight edge strip was cut wider than that on the lower arm. The distance from this edge to the center of the indicator-arm collar was measured accurately (12.33"), giving length "R". With the microscope focused sharply on this edge, the other sight edge strip was brought into focus by bending it slightly.

The torque arm was aligned so that the silk threads transmitting the forces from the torque weight pans are perpendicular to the lever in both horizontal and vertical planes. The pulleys were adjusted in both vertical and horizontal planes by moving pulley clamps on the support rods, as shown in Figure II. When alignment of the torque system was obtained, the torque arm and pulley support clamps were locked in place with their set screws.

The desired normal load was placed on the weight pan and then applied to the specimen by cranking down the weight release.

The zero reading of the indicator arms was then recorded.

By adding equal weights to the two weight pans, a known torque was applied to the specimen and the resulting twist was measured on the optical micrometer. After each reading the torque was removed and the zero reading recorded.

After measuring the distance "L" between pin-point indentations in the specimen, these pits were marked so that they could be distinguished from marks made in the succeeding test.

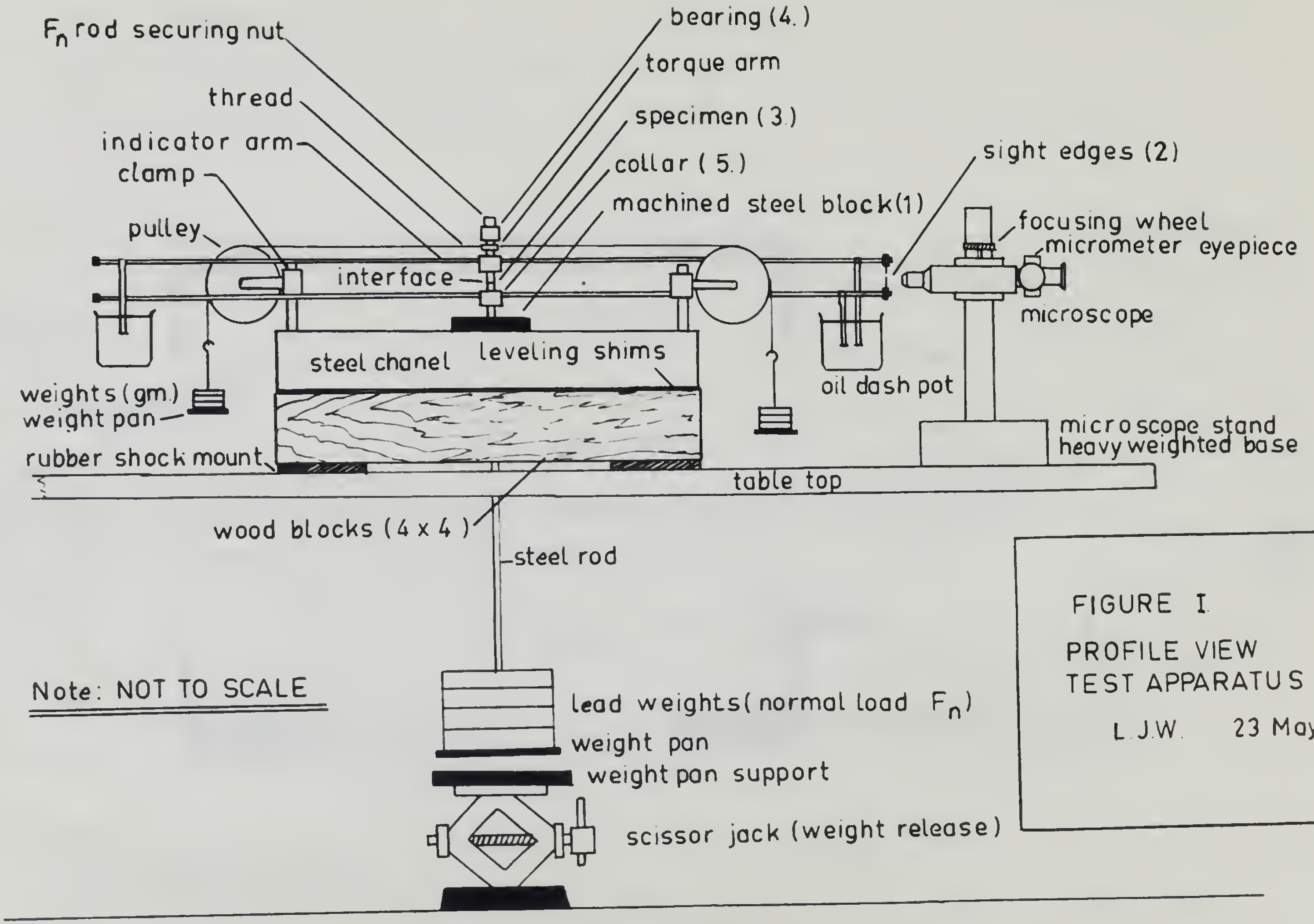


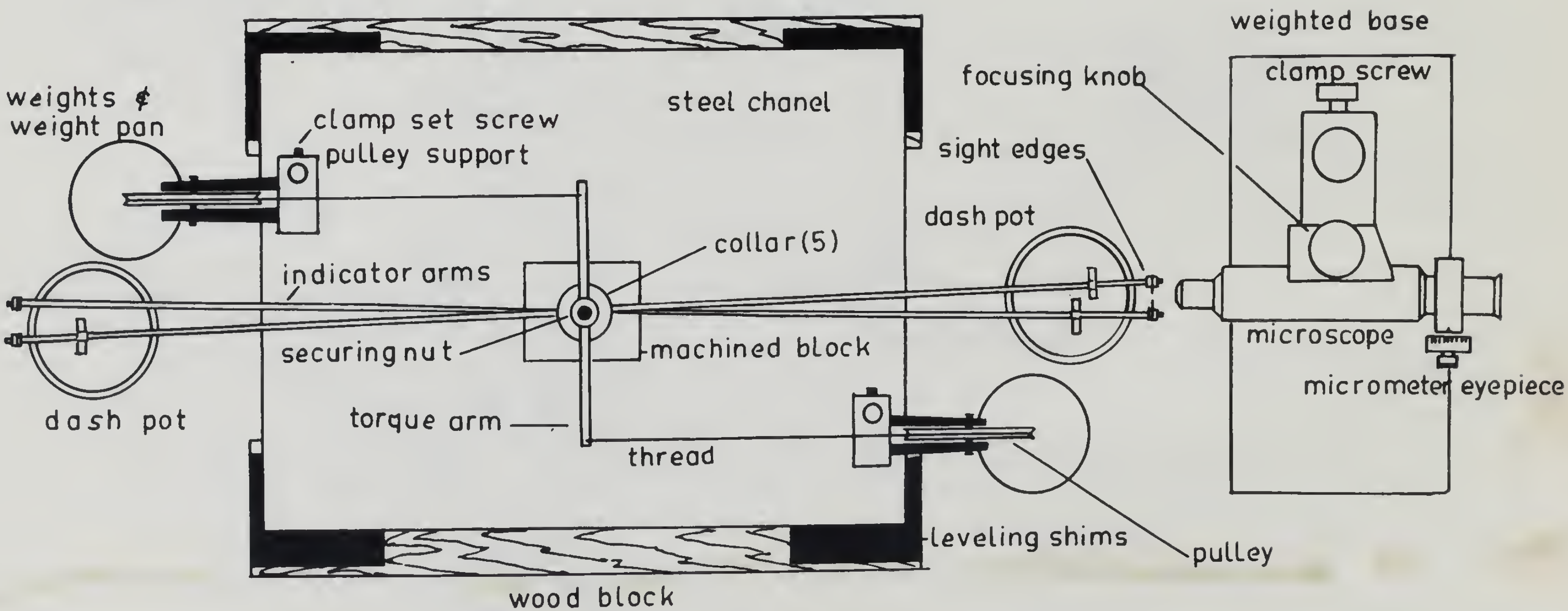
FIGURE I

PROFILE VIEW
TEST APPARATUS

L.J.W. 23 May, 1955

FIGURE II.

PLAN VIEW OF TEST APPARATUS



Note: NOT TO SCALE

L. J.W. 23 May, 1955

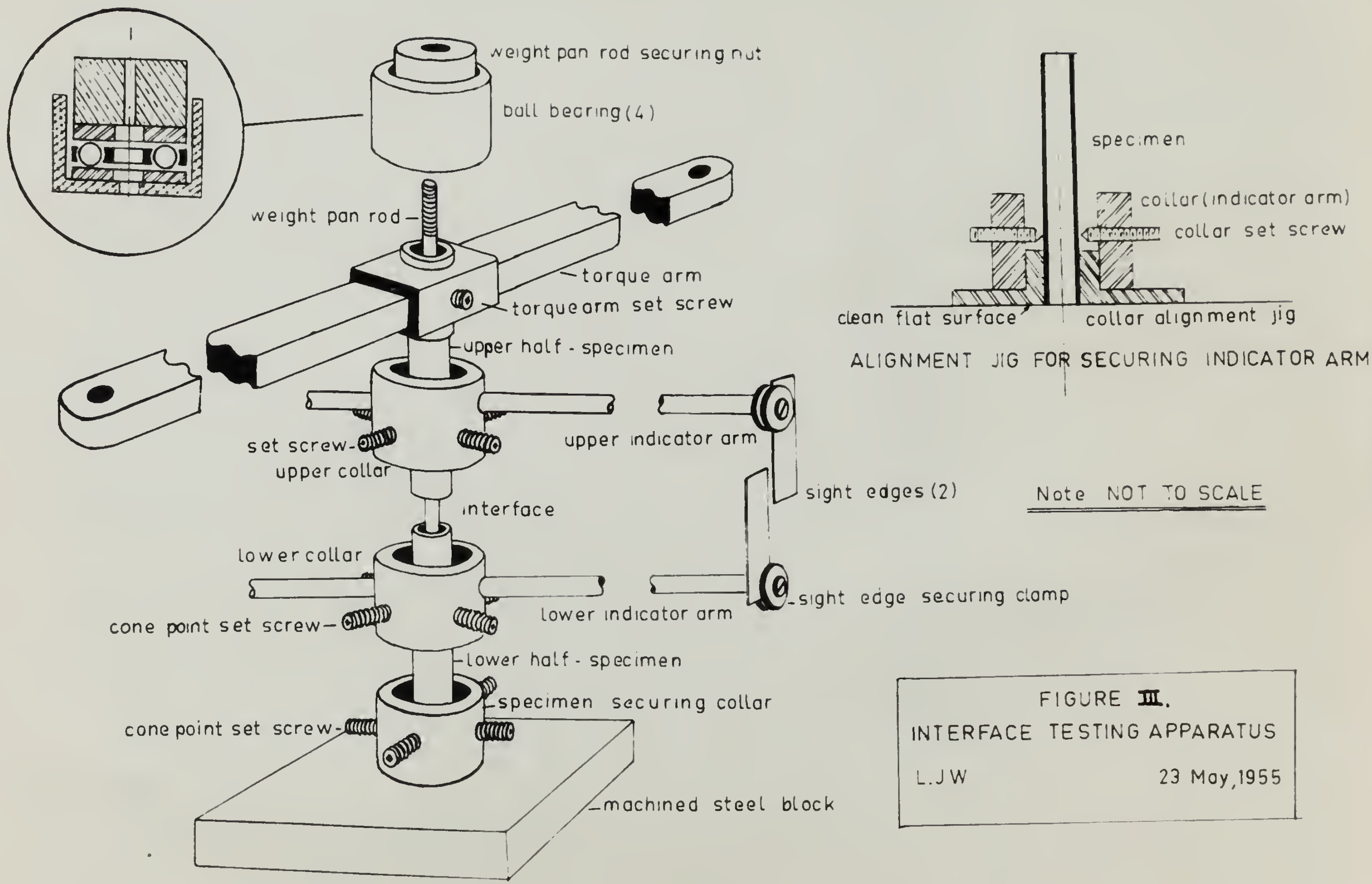


FIGURE III.
INTERFACE TESTING APPARATUS
L.J.W 23 May, 1955

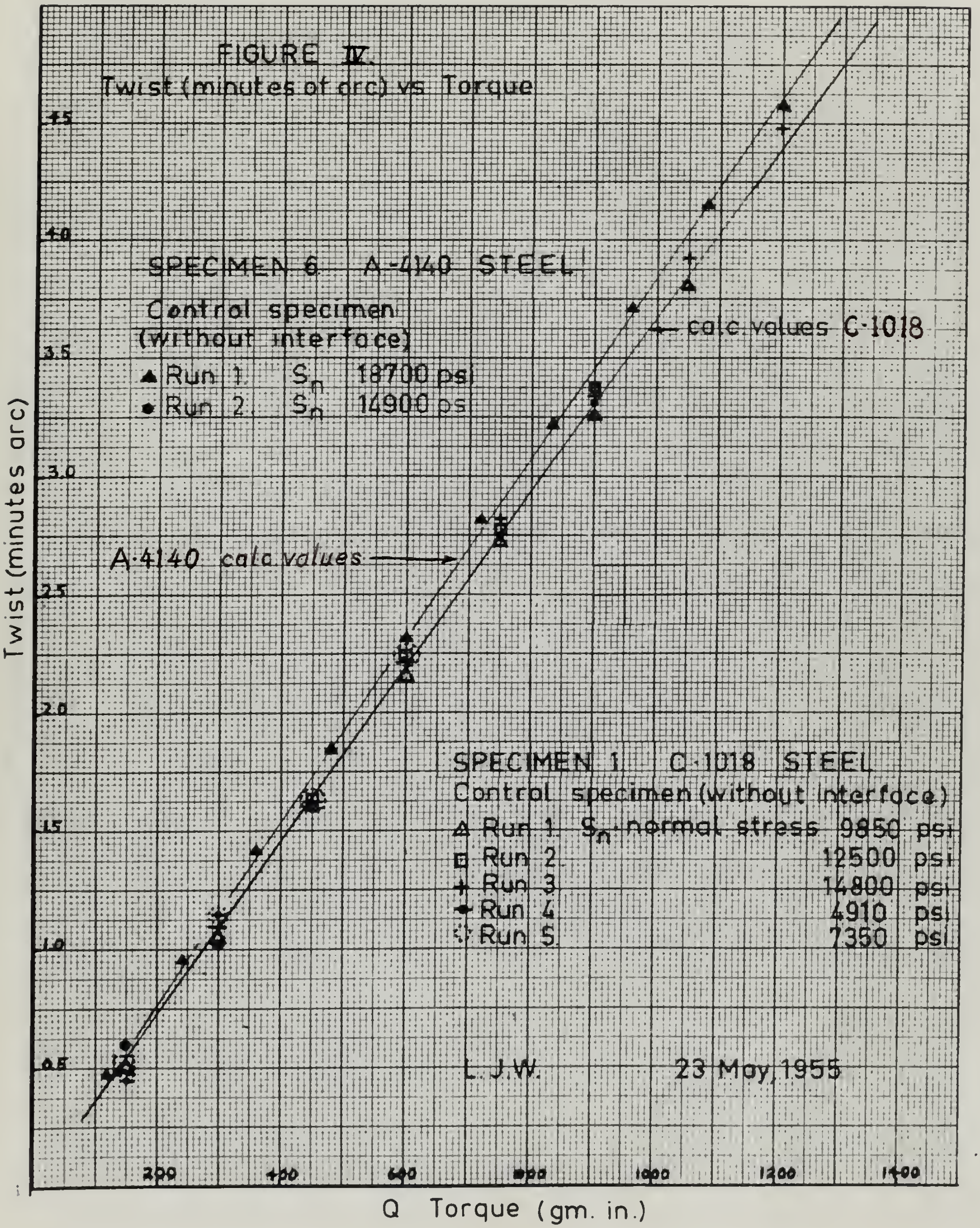
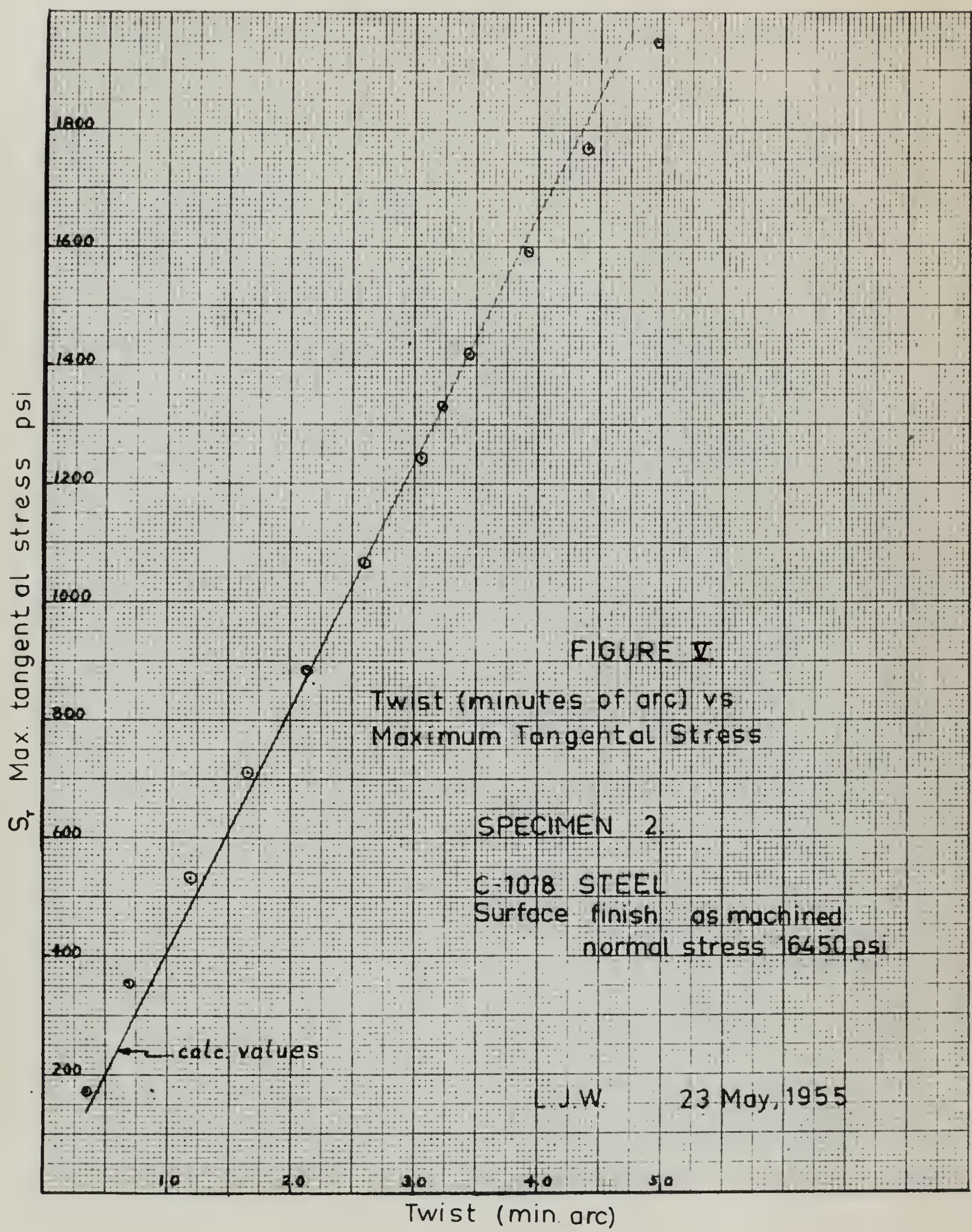
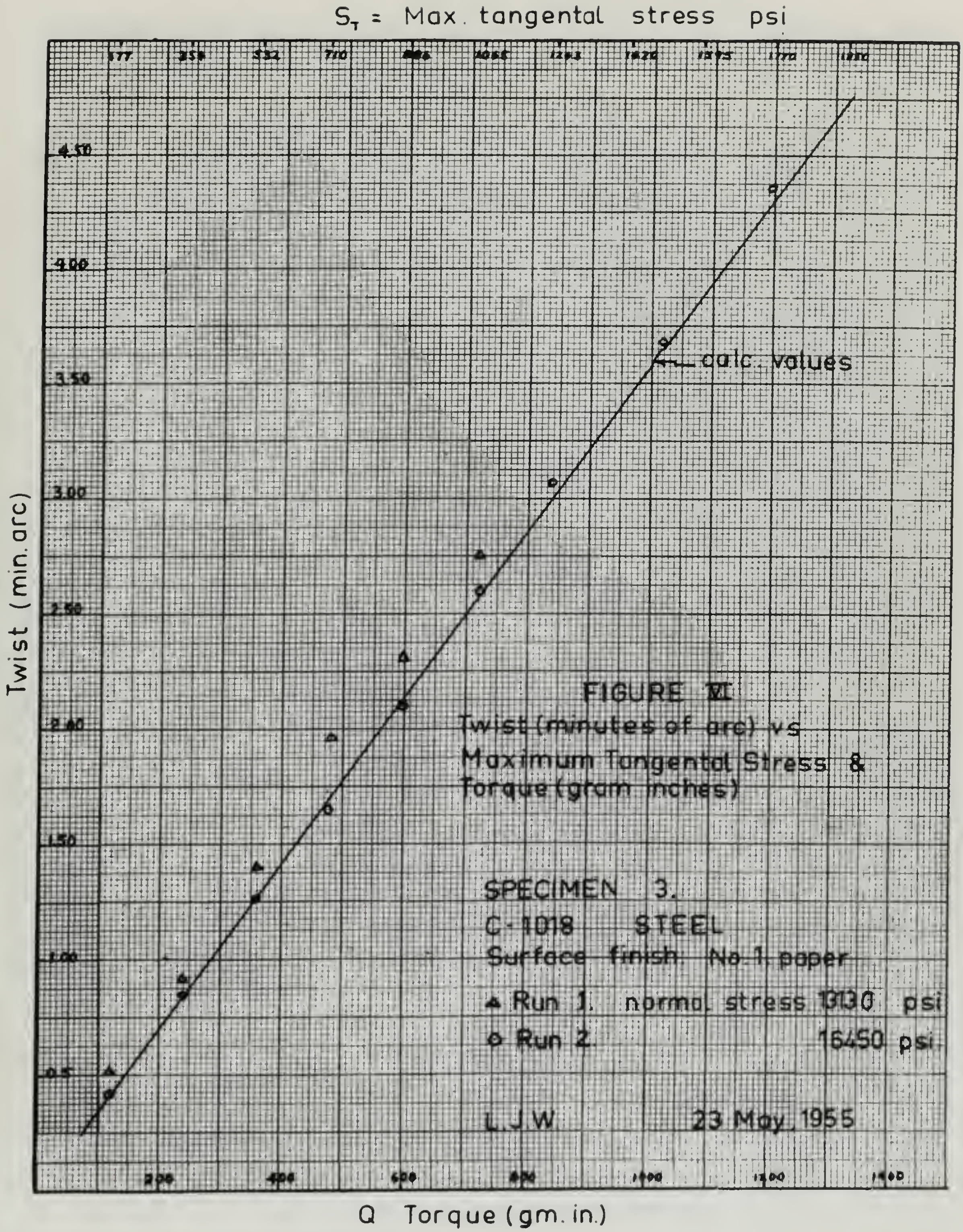
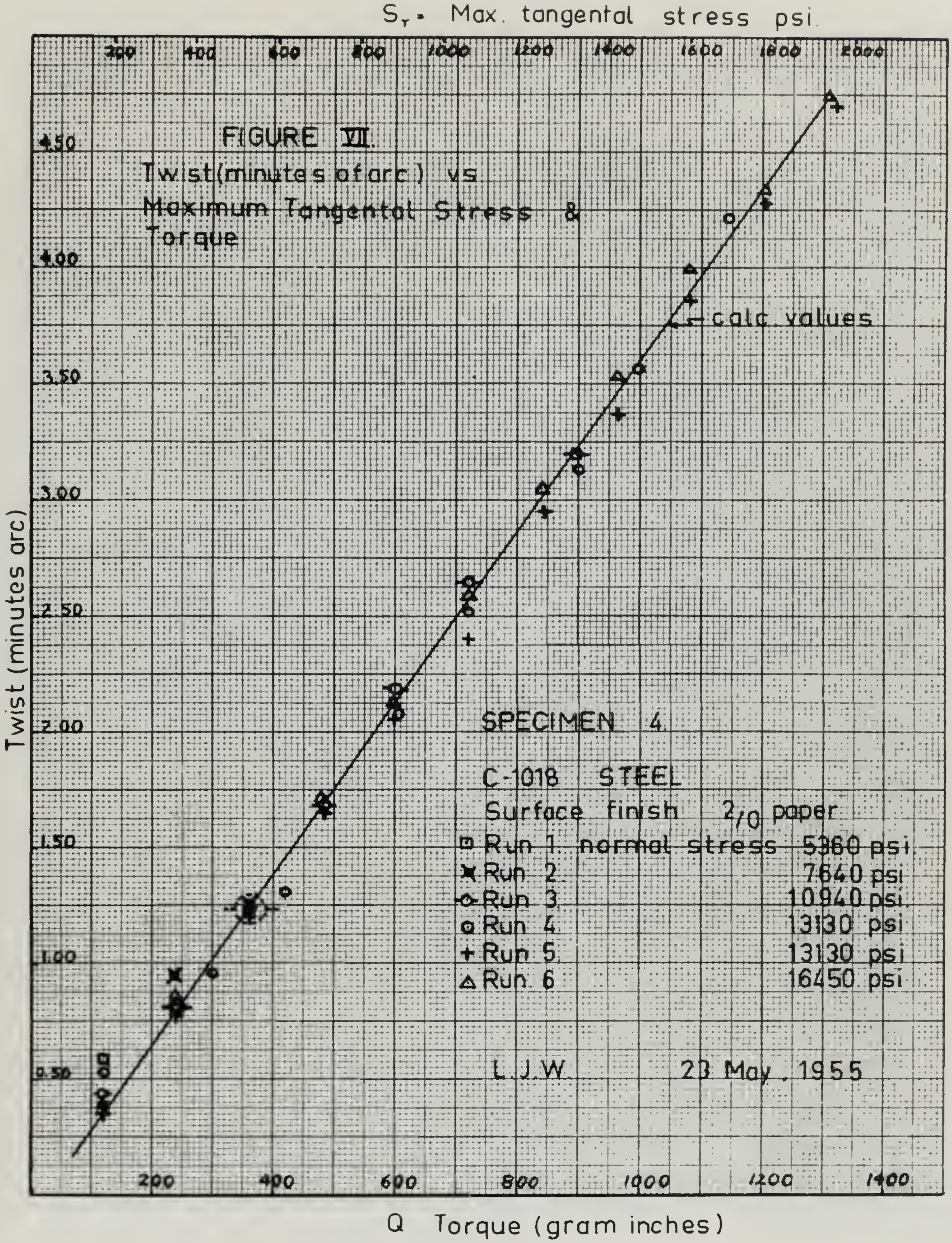


FIGURE IV.
Twist (minutes of arc) vs Torque
SPECIMEN 6 A-4140 STEEL
Control specimen
(without interface)
▲ Run 1 S_n 18700 psi
● Run 2 S_n 14900 psi
calc. values C-1018
A-4140 calc. values
SPECIMEN 1 C-1018 STEEL
Control specimen (without interface)
△ Run 1 S_n normal stress 9850 psi
□ Run 2 12500 psi
+ Run 3 14800 psi
Run 4 4910 psi
Run 5 7350 psi
L. J. W.
23 May 1955
Twist (minutes arc)
Q Torque (gm. in.)
200
400
600
800
1000
1200
1400
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5

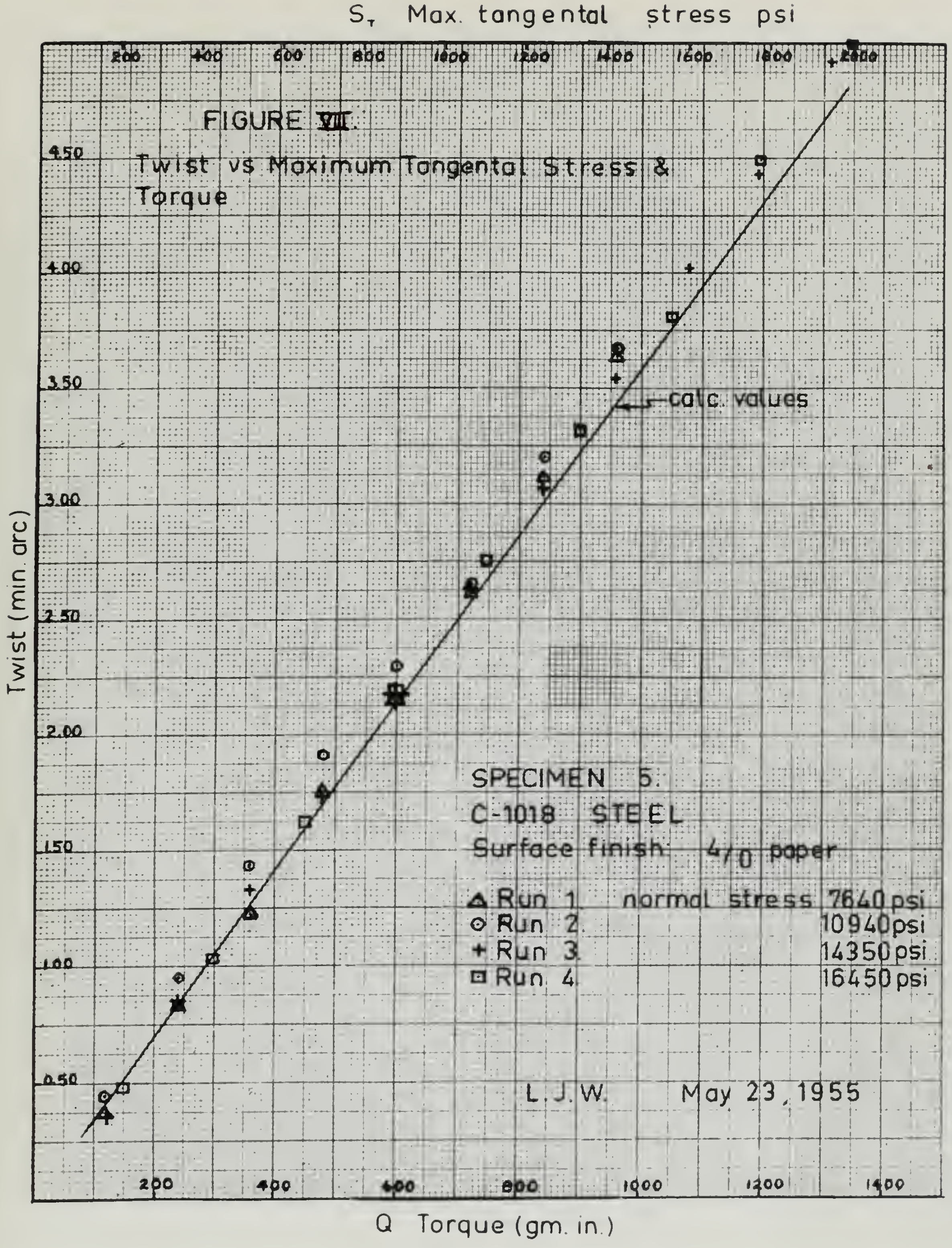


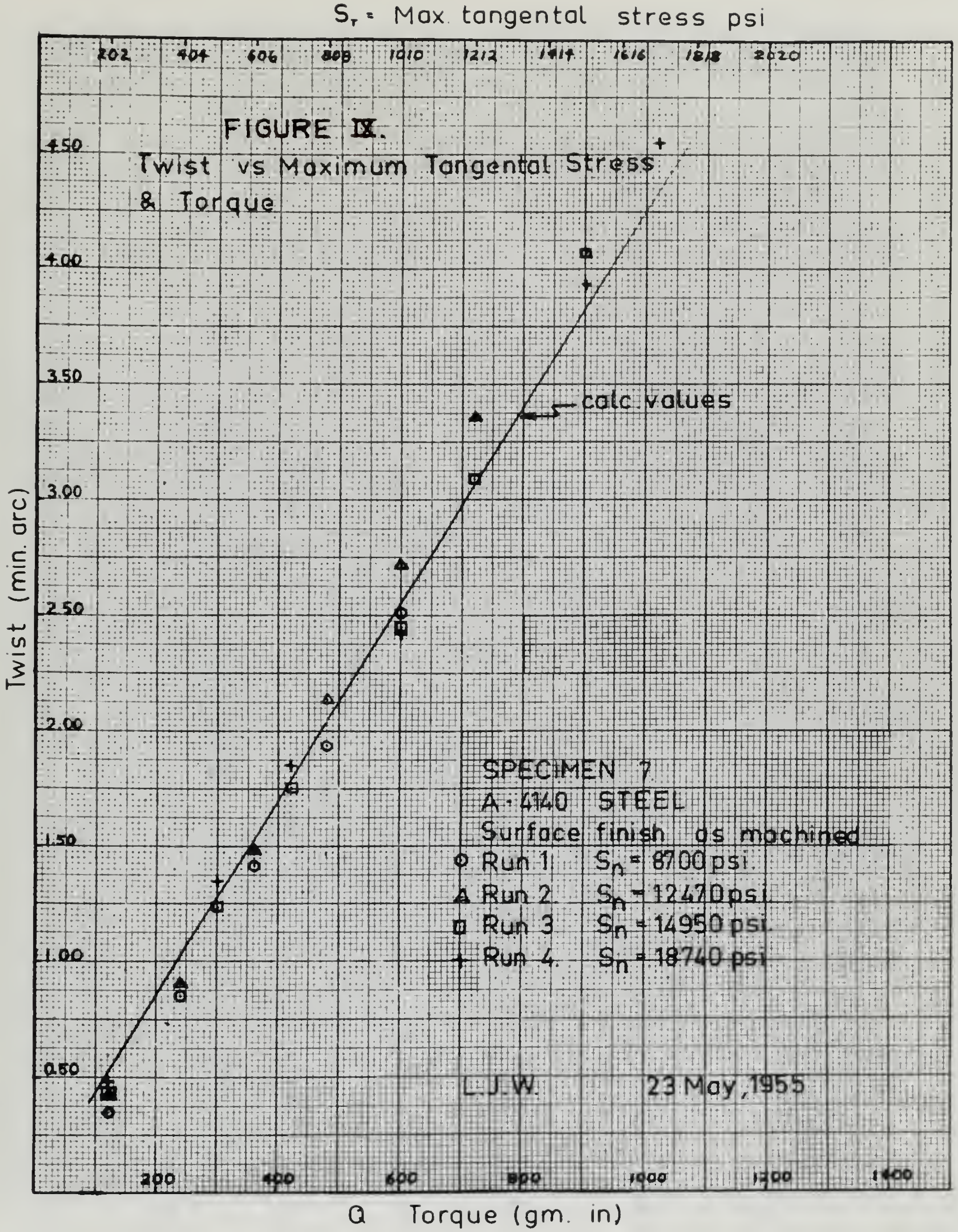
1800
1600
1400
1200
1000
800
600
400
200
S_τ Max. tangent al stress psi
1.0
2.0
3.0
4.0
5.0
Twist (min. arc)
calc. values
FIGURE V
Twist (minutes of arc) vs
Maximum Tangental Stress
SPECIMEN 2
C-1018 STEEL
Surface finish as machined
normal stress 16450 psi
L. J.W. 23 May, 1955


S_T = Max. tangental stress psi
calc. values
Twist (min. arc)
FIGURE VI
Twist (minutes of arc) vs
Maximum Tangental Stress &
Torque (gram inches)
SPECIMEN 3.
C-1018 STEEL
Surface finish No. 1 paper
▲ Run 1. normal stress 13130 psi
o Run 2. 16450 psi
L.J.W 23 May 1955
Q Torque (gm. in.)
200
400
600
800
1000
1200
1400
4.50
4.00
3.50
3.00
2.50
2.00
1.50
1.00
0.5



S_T = Max. tangental stress psi
200
400
600
800
1000
1200
1400
1600
1800
2000
FIGURE VII.
Twist (minutes of arc) vs
Maximum Tangental Stress &
Torque
calc. values
Twist (minutes arc)
4.50
4.00
3.50
3.00
2.50
2.00
1.50
1.00
0.50
SPECIMEN 4
C-1018 STEEL
Surface finish 2/0 paper
Run 1 normal stress 5360 psi
Run 2 7640 psi
Run 3 10940 psi
Run 4 13130 psi
Run 5 13130 psi
Run 6 16450 psi
L.J.W. 23 May, 1955
200
400
600
800
1000
1200
1400
Q Torque (gram inches)


S$_T$ Max. tangental stress psi
200
400
600
800
1000
1200
1400
1600
1800
2000
FIGURE VII.
Twist vs Maximum Tangental Stress & Torque
4.50
4.00
3.50
3.00
2.50
2.00
1.50
1.00
0.50
Twist (min. arc)
calc. values
SPECIMEN 5.
C-1018 STEEL
Surface finish: 4/0 paper
Run 1 normal stress 7640 psi
Run 2. 10940 psi
Run 3. 14350 psi
Run 4. 16450 psi
L. J. W.
May 23, 1955
200
400
600
800
1000
1200
1400
Q Torque (gm. in.)



S_T = Max. tangental stress psi
202 404 606 808 1010 1212 1414 1616 1818 2020
FIGURE IX.
Twist vs Maximum Tangental Stress & Torque
4.50
4.00
3.50
3.00
2.50
2.00
1.50
1.00
0.50
Twist (min. arc)
calc values
SPECIMEN 7
A-4140 STEEL
Surface finish as machined
Run 1 S_n = 8700 psi
Run 2 S_n = 12470 psi
Run 3 S_n = 14950 psi
Run 4 S_n = 18740 psi
L.J.W. 23 May, 1955
200 400 600 800 1000 1200 1400
Q Torque (gm. in)

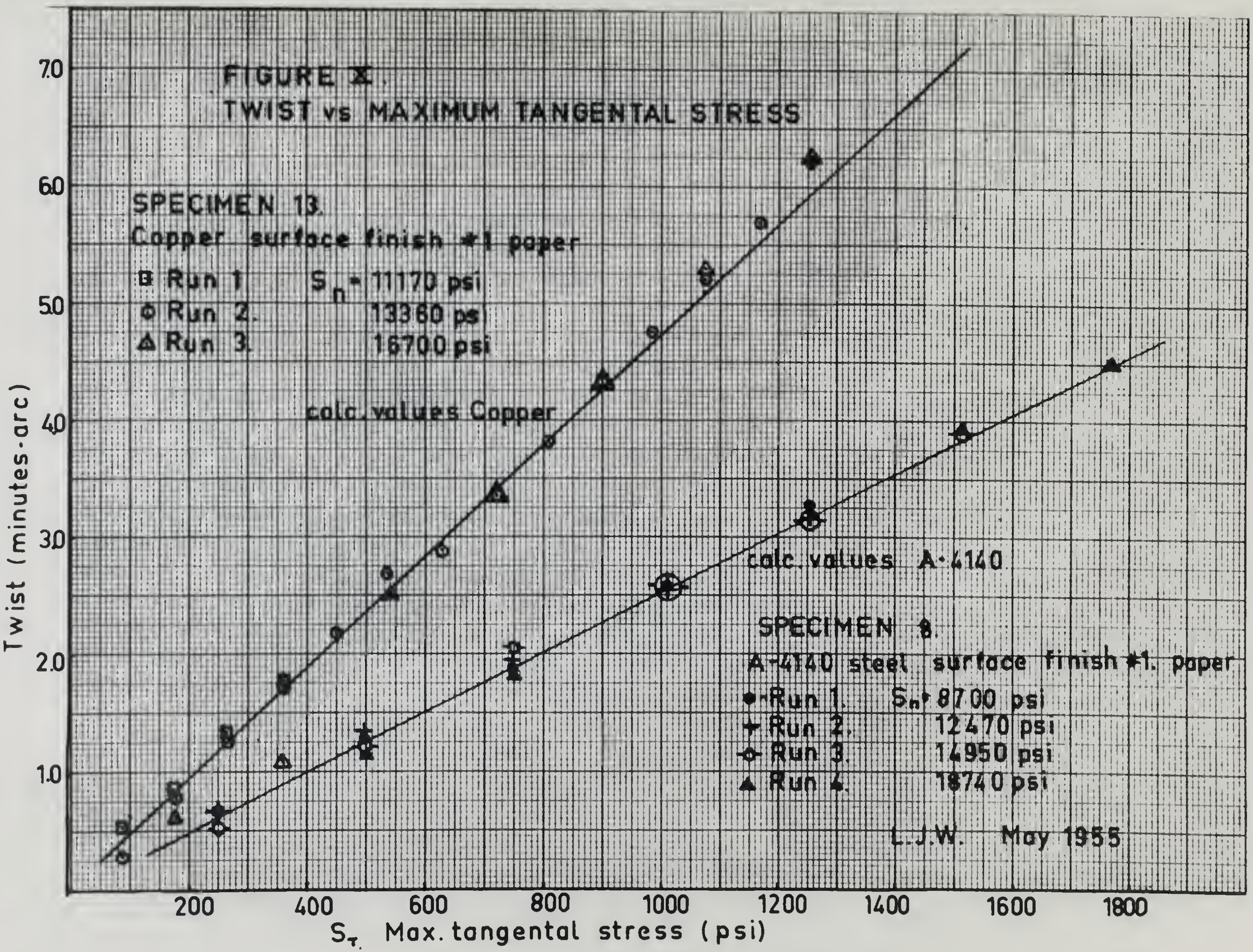


FIGURE X.
TWIST vs MAXIMUM TANGENTAL STRESS
SPECIMEN 13.
Copper surface finish #1 paper
Run 1. S_n = 11170 psi
Run 2. 13360 psi
Run 3. 16700 psi
calc. values Copper
calc. values A-4140
SPECIMEN 8.
A-4140 steel surface finish #1. paper
Run 1. S_n = 8700 psi
Run 2. 12470 psi
Run 3. 14950 psi
Run 4. 18740 psi
L.J.W. May 1955
Twist (minutes-arc)
7.0
6.0
5.0
4.0
3.0
2.0
1.0
200
400
600
800
1000
1200
1400
1600
1800
S_τ. Max. tangental stress (psi)

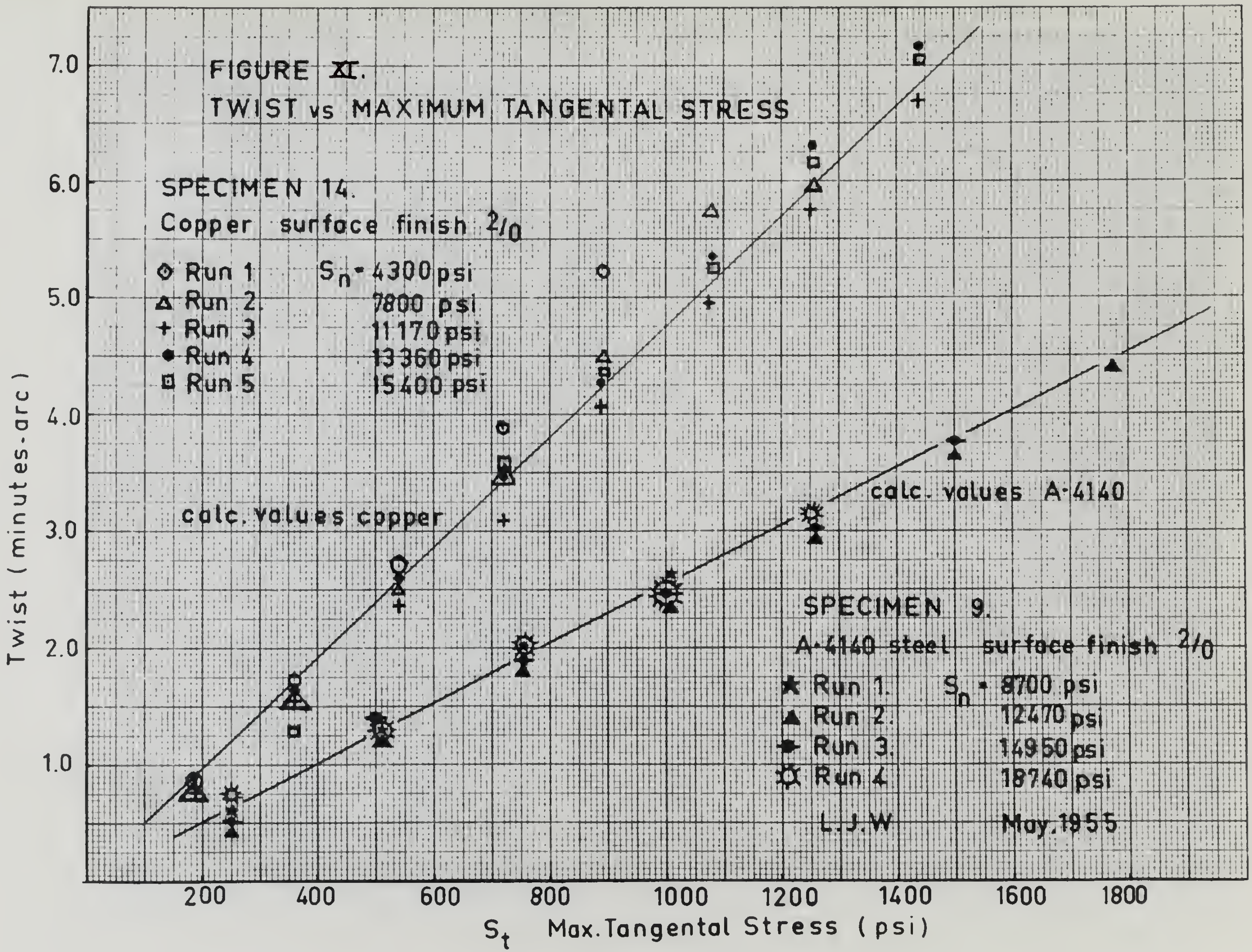


FIGURE XI.
TWIST vs MAXIMUM TANGENTAL STRESS
SPECIMEN 14.
Copper surface finish 2/0
Run 1 Sn = 4300 psi
Run 2. 7800 psi
Run 3 11170 psi
Run 4 13360 psi
Run 5 15400 psi
calc. values copper
calc. values A-4140
SPECIMEN 9.
A-4140 steel surface finish 2/0
Run 1. Sn = 8700 psi
Run 2. 12470 psi
Run 3. 14950 psi
Run 4 18740 psi
L.J.W May, 1955
Twist (minutes-arc)
7.0
6.0
5.0
4.0
3.0
2.0
1.0
200
400
600
800
1000
1200
1400
1600
1800
St Max. Tangental Stress (psi)

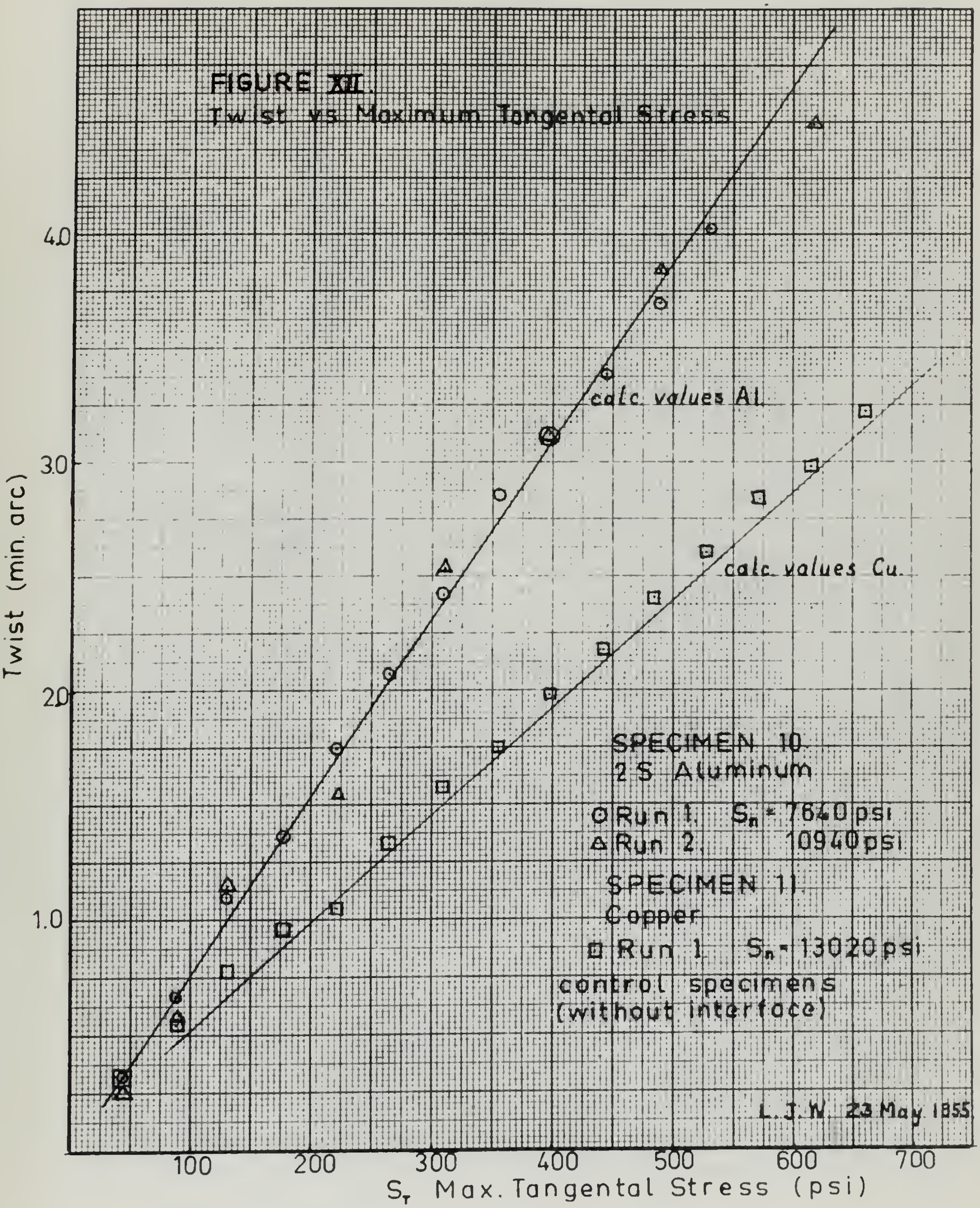


FIGURE XII.
Twist vs. Maximum Tangental Stress
calc. values Al.
calc. values Cu.
SPECIMEN 10.
2 S Aluminum
Run 1. Sn = 7640 psi
Run 2. 10940 psi
SPECIMEN 11.
Copper
Run 1. Sn = 13020 psi
control specimens
(without interface)
L. J. W. 23 May 1955
Twist (min. arc)
1.0
2.0
3.0
4.0
100
200
300
400
500
600
700
Sτ Max. Tangental Stress (psi)

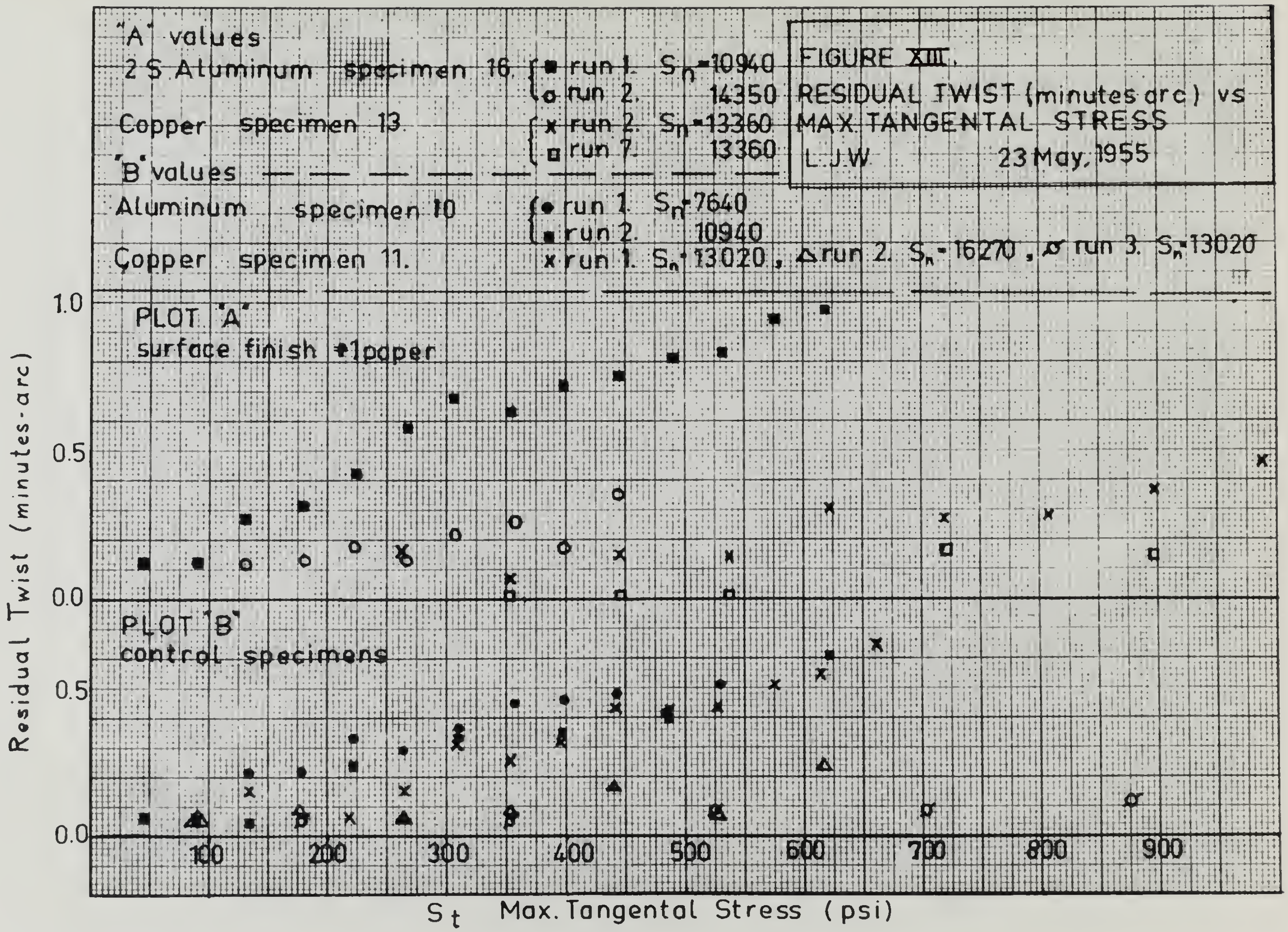


FIGURE XIII.
RESIDUAL TWIST (minutes arc) vs
MAX. TANGENTAL STRESS
L. J. W. 23 May, 1955
"A" values
2 S Aluminum specimen 16. ■ run 1. S_n=10940
o run 2. 14350
Copper specimen 13. x run 2. S_n=13360
□ run 7. 13360
"B" values
Aluminum specimen 10 ● run 1. S_n=7640
■ run 2. 10940
Copper specimen 11. x run 1. S_n=13020, Δ run 2. S_n=16270, ø run 3. S_n=13020
PLOT "A"
surface finish #1 paper
PLOT "B"
control specimens
Residual Twist (minutes-arc)
1.0
0.5
0.0
0.5
0.0
100
200
300
400
500
600
700
800
900
S_t Max. Tangental Stress (psi)

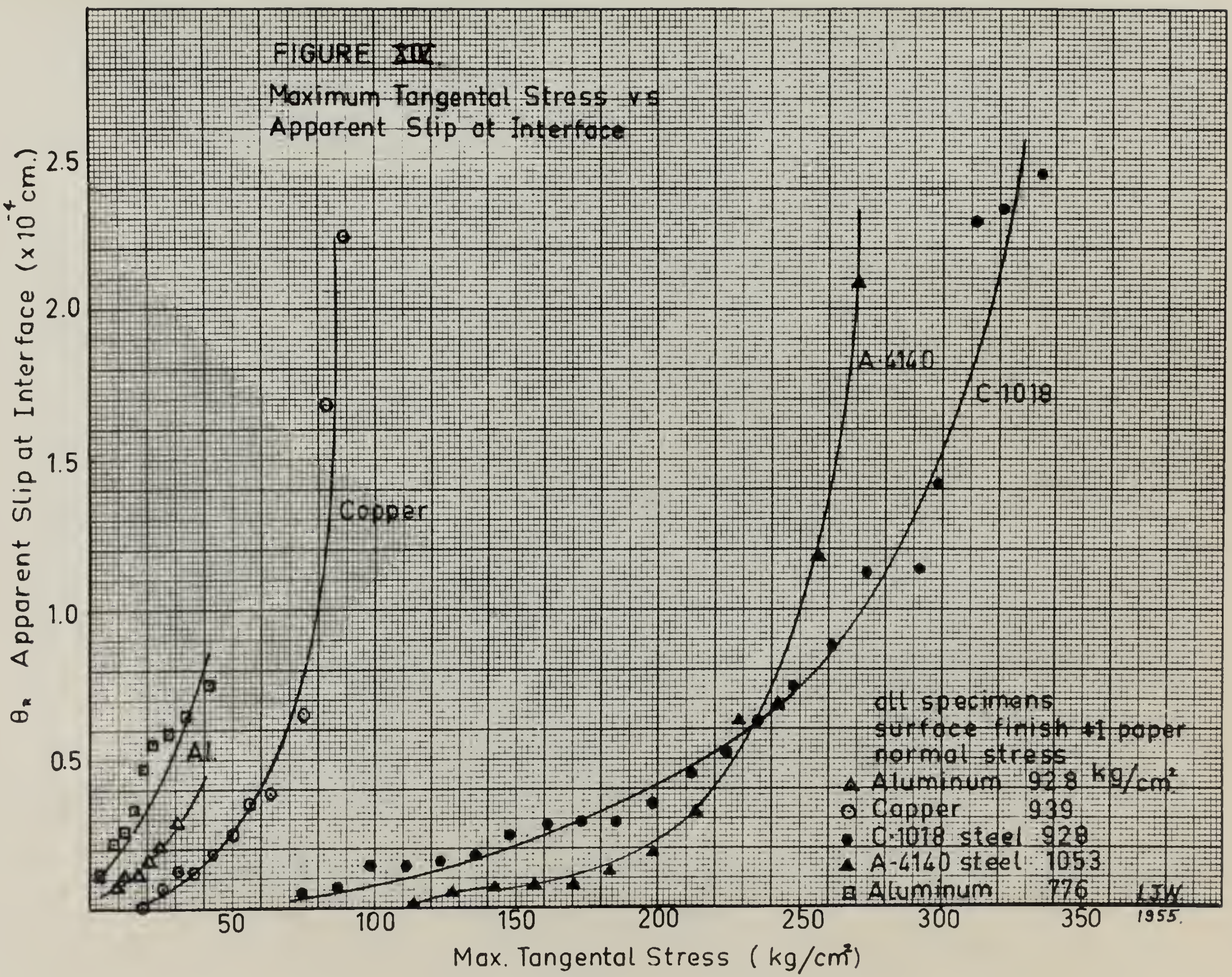


FIGURE XIV.
Maximum Tangental Stress vs
Apparent Slip at Interface

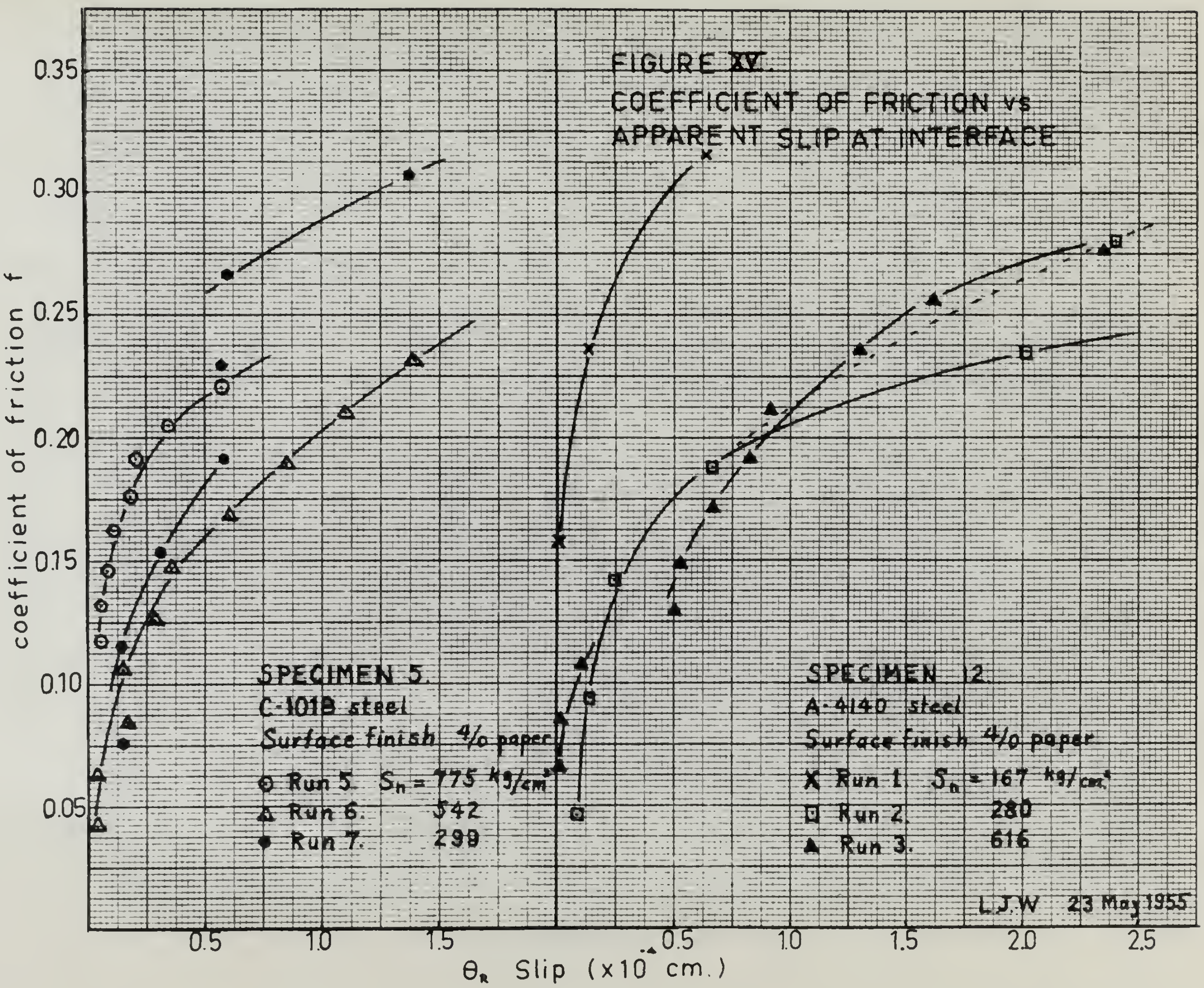


FIGURE XV.
COEFFICIENT OF FRICTION vs
APPARENT SLIP AT INTERFACE
coefficient of friction f
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.5
1.0
1.5
0.5
1.0
1.5
2.0
2.5
θR Slip (x10⁻⁴ cm.)
SPECIMEN 5.
C-1018 steel
Surface finish 4/0 paper
Run 5. Sn = 775 kg/cm²
Run 6. 542
Run 7. 299
SPECIMEN 12.
A-4140 steel
Surface finish 4/0 paper
Run 1. Sn = 167 kg/cm²
Run 2. 280
Run 3. 616
L.J.W. 23 May 1955

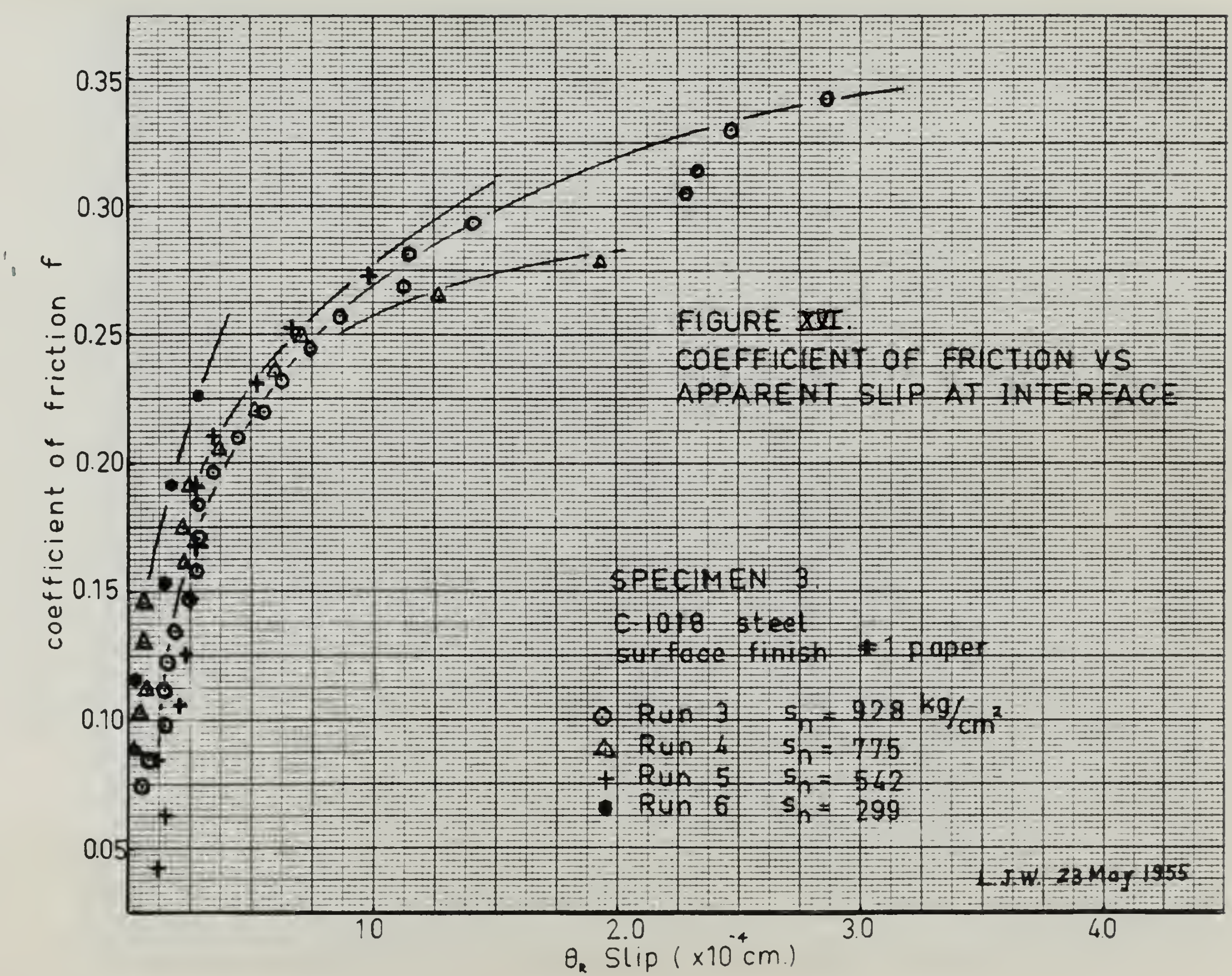


FIGURE XVI.
COEFFICIENT OF FRICTION VS
APPARENT SLIP AT INTERFACE
SPECIMEN 3
C-1018 steel
surface finish #1 paper
Run 3 s_n = 928 kg/cm²
Run 4 s_n = 775
Run 5 s_n = 542
Run 6 s_n = 299
L.J.W. 23 May 1955
0.35
0.30
0.25
0.20
0.15
0.10
0.05
coefficient of friction f
1.0
2.0
3.0
4.0
θ_R Slip (x10⁻⁴ cm.)

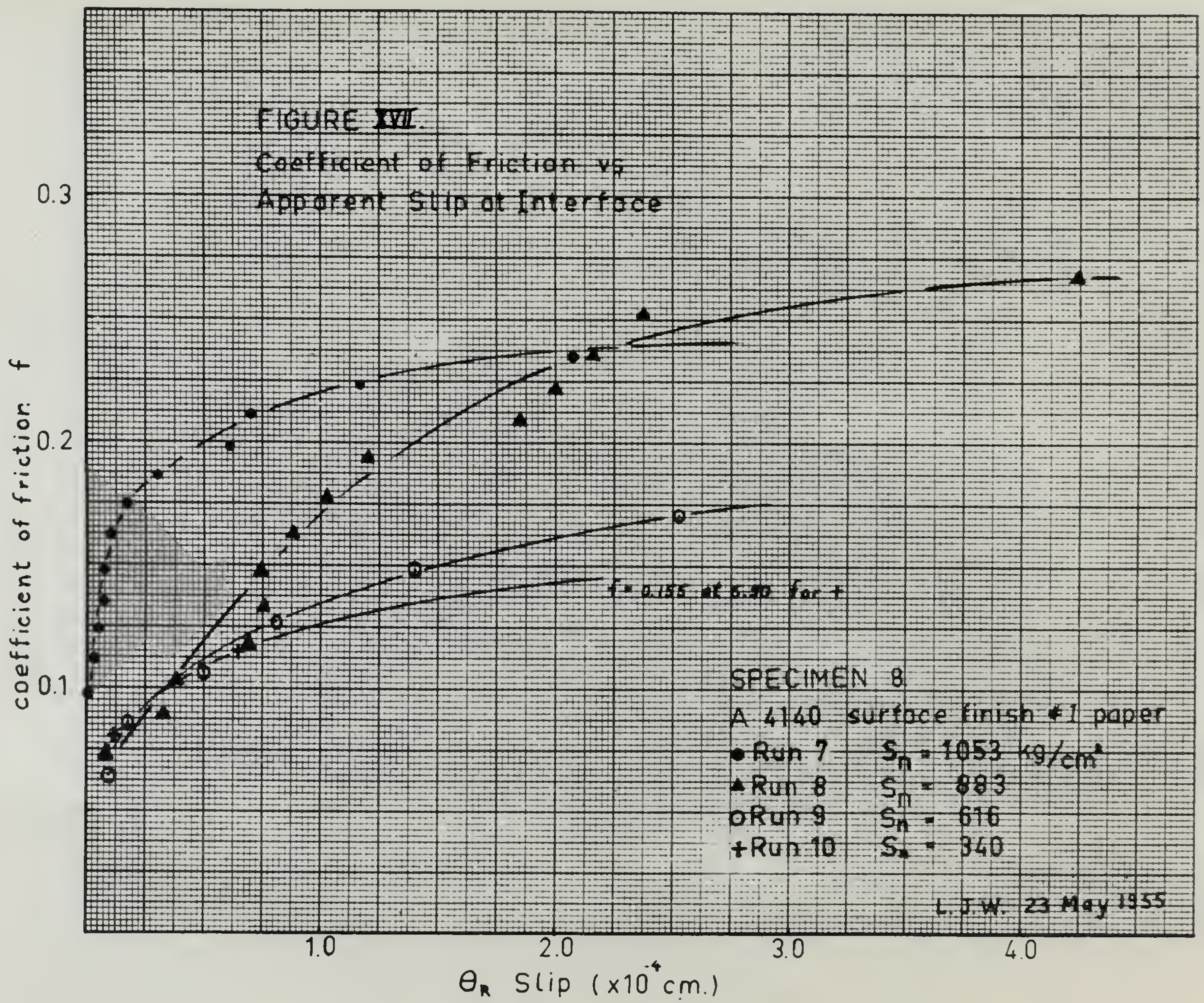


FIGURE XVII.
Coefficient of Friction vs
Apparent Slip at Interface
0.3
0.2
0.1
coefficient of friction f
1.0
2.0
3.0
4.0
θR Slip (x10-4 cm.)
f = 0.155 at 6.90 for +
SPECIMEN 8
A 4140 surface finish #1 paper
● Run 7 Sn = 1053 kg/cm²
▲ Run 8 Sn = 883
o Run 9 Sn = 616
+ Run 10 Sn = 340
L.J.W. 23 May 1955

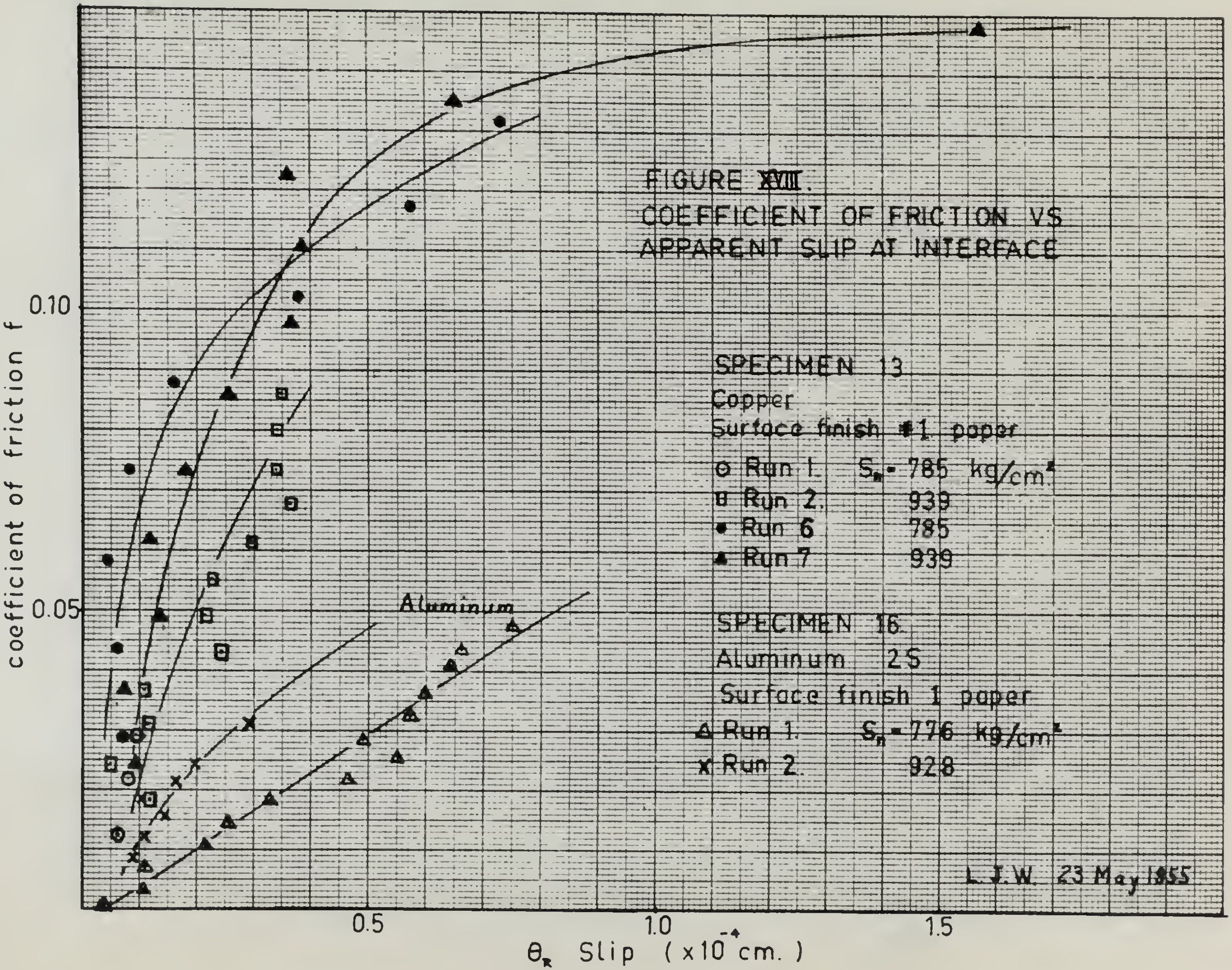


FIGURE XVIII.
COEFFICIENT OF FRICTION VS
APPARENT SLIP AT INTERFACE

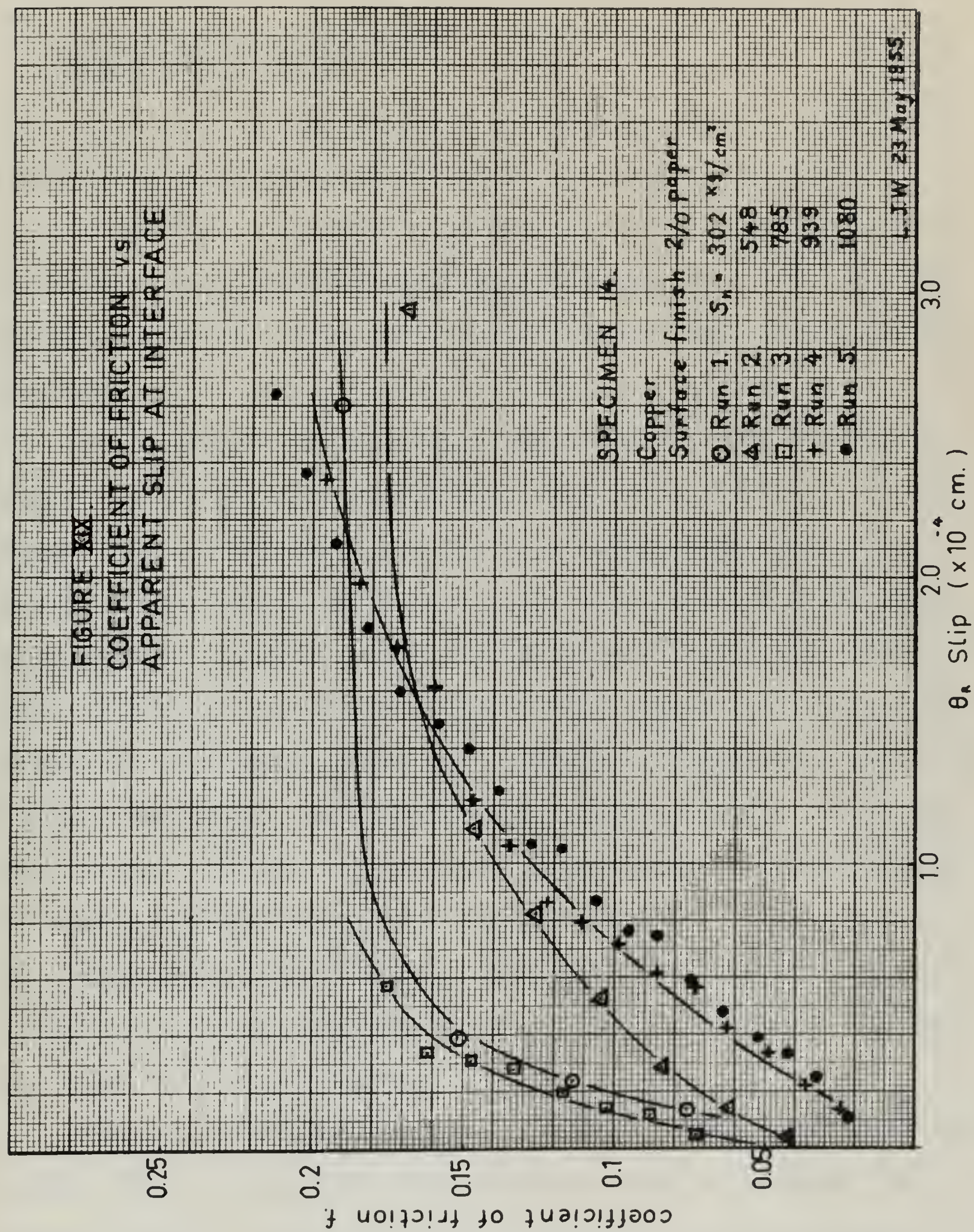


FIGURE XX
COEFFICIENT OF FRICTION vs
APPARENT SLIP AT INTERFACE
SPECIMEN 14
Copper
Surface finish 2/0 paper
Run 1 S_n = 302 kg/cm²
Run 2 548
Run 3 785
Run 4 939
Run 5 1080
L.J.W. 23 May 1955
coefficient of friction f
0.05
0.1
0.15
0.2
0.25
θ_A Slip (x10⁻⁴ cm.)
1.0
2.0
3.0

IV. DISCUSSION OF RESULTS

A. Initial Elastic Behavior of Metal Interfaces

Figures IV and XII compare observed deflections of the control specimens with computed deflections as predicted by elastic theory. All points plotted for the two steel control specimens are within the elastic range, as the test runs were terminated at the first indication of possible permanent deformation.

The values of shear modulus selected for the C-1018 and A-4140 steels were, respectively, 12×10^6 and 11.9×10^6 psi. Marks[6] gives the range of shear modulus values for all steels, excepting 18-8 stainless, as 11.0 to 11.9×10^6 psi. The computed angle of twist varies inversely with the value of shear modulus. Therefore it can be seen that, with the values of shear modulus selected, the computed deflections will be the minimum expected. As the observed values of twist agree with the minimum computed values, the possibility of excessive elastic twist in the control specimen is eliminated.

The shear modulus values for copper (hard-drawn tough pitch) and 2S Aluminum were given in the Handbook as 6.0 and 3.7×10^6 psi, respectively.

Figures V to VIII show the results of tests conducted with C-1018 steel specimens having various surface finishes and normal loads. Due to the close agreement between observed and

computed values of deflection for the control specimens, the observed angles of twist are compared with their corresponding theoretical value in this work.

The consistent agreement between test results for specimens with and without interfaces indicates that the interface has very little, if any, effect on the elastic twist.

All test runs with non-ferrous specimens were performed with the lower end of the specimen locked against movement. These test runs extended past the point at which the indicator failed to return to the initial position. The "B" plot of Figure XIII gives the observed values of residual twist recorded for the non-ferrous control specimen runs shown on Figure XII. Hard-drawn electrolytic tough pitch copper has a yield strength of 40,000 psi with 0.5% extension under load. Three successive runs on the copper control specimen show this effect clearly. The recorded values of deformation decreased with each run. Plot "A" of this figure shows the residual twist recorded for similar runs conducted on specimens with a No.1 surface finish. The effects contributed by the metal interface can easily be obtained by subtraction of the plotted values for any two corresponding tests. The point at which slip occurs on the copper specimen is easily noted, as the residual twist value increased sharply at that point.

Figures X and XI compare observed and calculated values of deflection for various test runs on copper and A-4140 steel specimens. These tests further substantiate the results recorded previously for C-1018 steel. Test runs using copper,

C-1018 steel, or A-4140 steel under various conditions of normal loading and with different degrees of surface finish failed to indicate any material contribution by the interface to elastic twist.

B. Initial Frictional Behavior of Metal Interfaces

Figure XIV compares the results of a similar test run conducted on four different metals. Comparing the results of the two steel specimens, it is seen that deformation occurred first in the C-1018 steel. This is attributed to yield in the asperities in the metal interface. C-1018 steel, with a yield strength of 48,000 psi as compared with 131,000 psi for the A-4140, would be expected to yield first. It should be noted that the deformation observed here is of the order of 0.000005 cm. The C-1018 curve crosses the A-4140 line at an apparent slip value of 0.00006 cm. This shows that sliding effects are now predominant, as the A-4140 steel with its smaller coefficient of friction slides more easily than the C-1018 steel.

Deformation in the two non-ferrous specimens commenced at fairly low values of tangential stress. It is interesting to note that the aluminum slid freely, and the copper curve rose sharply, at the same value of θ_R at which the two steel curves crossed. Again it appears that, up to this point, deformation was primarily due to yield in the material, while subsequent failure of the interfacial bonds permitted friction effects to predominate.

Figures XV to XIX indicate that the value of the friction coefficient increases with small increments of slip until the normally expected value is attained, at which point the curve flattens out and free sliding results.

The curves plotted in Figures XVI, XVII, and XIX are all fairly smooth, giving no indication of a transition from interfacial deformation to true slip. The plots on Figures XV and XVIII, in contrast, show prominent breaks and discontinuities in the region below 0.00005 cm of apparent slip. The apparent slip in these cases may be due to slip, deformation of the metal, or combinations of both effects. An accurate evaluation of slip with this apparatus is therefore impossible due to the problem of separating the true slip from the deformations produced by yield effects in the asperities in the metal interfaces.

In evaluating these results, some information on the accuracy of the readings is in order. A discussion of the accuracy of the apparatus follows in Appendix A, Details of Procedure.

$$\theta_R = \pm 1.80 \times 10^{-7} \text{ centimeters}$$

$$\phi_o = \pm 0.00229 \text{ minutes of arc}$$

V. CONCLUSIONS AND RECOMMENDATIONS

A. Conclusions - Elastic Behavior

1. The metal interface does not contribute materially to elastic twist.
2. Elastic deflections of a metal specimen with an interface agree with values computed by elastic theory for a continuous specimen.
3. The deflections predicted by elastic theory are obtained regardless of metal, surface finish, or normal stress selected. (Normal loads applied were selected so that resulting normal stresses were safely below critical range which would cause buckling or compressive failure of the column.)
4. The predicted elastic deflections are obtained in non-ferrous metals even though extension of the metal under load resulted in small concurrent permanent deformations.

B. Conclusions - Frictional Behavior

1. The value of the coefficient of friction was initially low for extremely small values of slip at the interface.
2. The value of the coefficient increased with small increments of slip up to the normally expected value at which point free sliding resulted.

3. An exact assessment of the increments of slip at the interface was impossible with this experimental technique, due to small amounts of deformation contributed by yield in the asperities in the metal interface. Notwithstanding this factor, the trend indicated that the coefficient of friction starts from zero for zero values of slip and applied stress, and increases with small increments of slip until the normal value is reached.

C. <u>Recommendations</u>

Due to the consistency of the results, both quantitatively and qualitatively, further investigation of metal interfaces for abnormal elastic twist is not recommended.

This apparatus was not designed for investigation of frictional behavior in the slow speed range; therefore modifications of the equipment are deemed necessary prior to any further investigations of the stick slip phenomena with this basic experimental arrangement.

APPENDIX A

DETAILS OF PROCEDURE

DETAILS OF PROCEDURE

In the redesign of the test apparatus, various schemes for measuring angular twist of the specimen were considered. Selection of an optical lever in lieu of the metal indicator arms of the original apparatus was rejected due to the length of light beam required. This would require excessive floor space for the apparatus or would involve the use of a complex prism system. Distribution of the equipment over a large floor space would present a problem in isolating the apparatus from vibrations imparted by the building structure. Another source of error would be encountered in the resolution of the light image on the measuring scale.

The possibility of utilizing inferometry was investigated. Measurement of exceedingly small angles of twist could be obtained by this means, but the cost and complexity of the apparatus required for this work were greater than warranted.

It was decided that a sufficient increase in the sensitivity of the system could be obtained by use of better materials, measuring equipment, and refinements in the experimental method.

The original apparatus utilized an apparatus having an indicator arm length of 7.5" and an optical vernier in which one scale unit represented 0.001 inch of indicator movement.

Estimating the movement of one indicator arm to $\pm$ 0.1 scale unit, the resulting accuracy with two arms would be $\pm$ 0.2 scale unit.

$$\psi_o = \frac{\psi_s}{R} \times \frac{180}{\pi} \times 60 = \text{observed twist in minutes of arc.}$$

Therefore accuracy of the original apparatus was

$$\psi_o = \pm \frac{0.0002}{7.5} \times 3440 = \pm 0.092 \text{ minutes of arc.}$$

By the use of a microscope equipped with a 10 x objective and an optical micrometer, the accuracy was increased considerably. The use of the light-weight but rigid tubular indicator arms permitted increasing the value of R to 12.328".

Calibration of the optical micrometer against a micrometer stage showed that one micrometer drum unit represented 0.00004101 inch of indicator movement. Estimating the micrometer readings to ± 0.1 drum unit would result in an accuracy in observed deflections of ± 0.2 drum units. Therefore, accuracy of the present apparatus is

$$\psi_o = \pm \frac{0.000008202}{12.33} \times 3440 = \pm 0.002290 \text{ minutes of arc}$$

The value of apparent slip (θ_R) measured with this apparatus is computed as follows: For the C-1018 steel specimen with the outside diameter of 0.2365" and inside diameter of 0.191", the mean radius (r_m) of the specimen is 0.1069".

Therefore

$$\frac{\theta_R}{r_m} = \pm \frac{0.000008202}{12.33}$$

$$\theta_R = \pm 7.10 \times 10^{-8} \text{ inches}$$

$$\theta_R = \pm 1.80 \times 10^{-7} \text{ centimeters}$$

APPENDIX B

SUMMARY OF DATA AND CALCULATIONS

SPECIMEN NO.1

Material: C-1018 Steel

Control Specimen:
(without interface)

$D_o = 0.2365"$
$D_i = 0.191"$
$L = 1.0156"$

$D_o^2 - D_i^2 = 0.01945$

$D_o^4 - D_i^4 = 0.001798$

$A = \frac{\pi}{4}(D_o^2 - D_i^2) = 0.01526$

$S_N = \frac{F_N}{A} = \frac{F_N}{0.01526}$ psi

$S_T = 1.121 \times 10^{-2} \frac{D_o}{D_o^4 - D_i^4} \times Q_A$

$S_T = 1.475 \times Q_A$ psi

$\psi_c = \frac{77.3 \times L \times Q_A}{(D_o^4 - D_i^4) \times E_s}$

$E_s = 12 \times 10^6$ psi

$\psi_c = 0.00364\ Q_A$ min.arc.

$\psi_o = 0.01144\ \psi_s$ min.arc.

			Run No.1	Run No.2
			F_N = 150 lb.	F_N = 190.7 lb.
			S_N = 9850 psi	S_N = 12500 psi
Q_A	S_T	ψ_c	ψ_o	ψ_o
150	221.5	0.547	0.504	0.504
300	443	1.093	1.053	1.018
450	665	1.640	1.590	1.635
600	886	2.185	2.165	2.250
750	1107	2.733	2.735	2.776
900	1330	3.280	3.265	3.380
1050	1550	3.830	3.800	- -
1200	1770	4.370	- -	- -

SPECIMEN NO.1 (continued)

			<u>Run No.3</u>	<u>Run No.4</u>	<u>Run No.5</u>
			F_N = 225.4 lb.	F_N = 74.91 lb.	F_N = 111.9 lb.
			S_N = 14800 psi	S_N = 4910 psi	S_N = 7350 psi
Q_A	S_T	ψ_c	ψ_o	ψ_o	ψ_o
150	221.5	0.547	0.539	0.457	0.515
300	443	1.093	1.098	1.155	1.109
450	665	1.640	1.670	- -	1.647
600	886	2.185	2.210	- -	2.240
750	1107	2.733	2.820		
900	1330	3.280	3.345		
1050	1550	3.830	3.920		
1200	1770	4.370	4.480		

Specimen No.2

Material: C-1018 Steel

Surface Finish:
As Machined (Lathe)

Specimen No.3

Material: C-1018 Steel

Surface Finish:
No.1 Emery Paper

$D_o = 0.2365"$

$D_i = 0.191"$

$L = 1.0078"$

$D_o^2 - D_i^2 = 0.01945$

$D_o^4 - D_i^4 = 0.001798$

$A = \frac{\pi}{4}(D_o^2 - D_i^2) = 0.01526 \text{ in}^2$

$S_N = \frac{F_N}{A} = \frac{F_N}{0.01526}$ psi

$S_T = 1.121 \times 10^{-2} \frac{D_o}{D_o^4 - D_i^4} \times Q_A$

$S_T = 1.475 \times Q_A$ psi

$\phi_c = \frac{77.3 \times L \times Q_A}{(D_o^4 - D_i^4) \times E_s}$

$\phi_c = 0.00362\ Q_A$ min.arc.

$\phi_o = 0.01144\ \phi_s$ min.arc.

			SPECIMEN NO.2	SPECIMEN NO.3	
			Run No.1	Run No.1	Run No.2
			F_N = 251.2 lb.	F_N = 200.3 lb.	F_N = 251.2 lb.
			S_N = 16450 psi	S_N = 13130 psi	S_N = 16450 psi
Q_A	S_T	ϕ_c	ϕ_o	ϕ_o	ϕ_o
120	177	0.435	0.355	0.435	0.413
240	354	0.8695	0.687	0.918	0.849
360	532	1.304	1.180	1.410	1.260
480	710	1.740	1.662	1.925	1.650
600	886	2.177	2.145	2.315	2.100
720	1065	2.608	2.600	2.760	2.590
840	1243	3.043	3.050	- -	3.074
900	1330	3.260	3.234	- -	- -
960	1420	3.480	3.440	- -	- -
1080	1595	3.910	3.920	- -	- -
1200	1770	4.350	4.400	- -	4.400
1320	1950	4.780	4.970	- -	- -

SPECIMEN NO. 3

Run No.3

F_N = 201.29 lb.

F_N = 41.3 kg

S_N = 13200 psi

s_n = 928 kg/cm^2

G_A	s_t	ψ_R	θ_R	f	G_A	s_t	ψ_R	θ_R	f
240	24.9	- - -	- - -	0.0245	1920	199.0	0.459	36.20 x 10^{-6}	0.1960
360	37.4	- - -	- - -	0.0368	2040	211.5	0.573	45.20 "	0.2083
480	49.9	- - -	- - -	0.0490	2160	224.0	0.641	50.60 "	0.2206
600	62.3	- - -	- - -	0.0613	2280	236.5	0.802	63.30 "	0.2326
720	74.9	0.057	4.50 x 10^{-6}	0.0736	2400	248.7	0.940	74.15 "	0.2450
840	87.5	0.080	6.31 "	0.0859	2520	261.2	1.110	87.50 "	0.2575
960	99.9	0.195	15.38 "	0.0982	2640	273.5	1.420	112.0 "	0.2695
1080	112.0	0.183	14.42 "	0.1103	2760	293.0	1.432	113.0 "	0.2820
1200	124.3	0.206	16.25 "	0.1225	2880	298.5	1.787	141.0 "	0.2940
1320	137.0	0.229	18.06 "	0.1350	3000	311.0	2.910	229.5 "	0.3065
1440	149.4	0.321	25.30 "	0.1470	3120	323.0	2.945	232.4 "	0.3186
1560	161.6	0.344	27.12 "	0.1593	3240	336.0	3.095	244.0 "	0.3310
1680	174.3	0.355	28.00 "	0.1715	3360	348.5	3.660	288.5 "	0.3432
1800	186.6	0.344	27.12 "	0.1840	3480	361.0	- - -	- - - - - - -	0.3560

SPECIMEN NO.3 (continued)

<u>Run No.4</u>

F_N = 168.16 lb.

F_N = 76.4 kg

S_N = 11020 psi

s_n = 775 kg/cm^2

C_A	ψ_R	θ_R	f
240	- - -	- - -	0.0293
360	- - -	- - -	0.0440
480	- - -	- - -	0.0587
600	- - -	- - -	0.0734
720	0.023	1.81 x 10^{-6}	0.0880
840	0.057	4.50 "	0.1026
960	0.092	7.25 "	0.1172
1080	0.080	6.31 "	0.1320
1200	0.080	6.31 "	0.1466
1320	0.344	27.12 "	0.1613
1440	0.344	27.12 "	0.1760
1560	0.321	25.30 "	0.1906
1680	0.470	37.08 "	0.2050
1800	0.665	52.45 "	0.2200
1920	0.768	60.60 "	0.2345
2040	0.894	70.50 "	0.2492
2160	1.625	128.0 "	0.2640
2280	2.450	193.2 "	0.2785
2400	- - -	- - -	0.2935

SPECIMEN NO.3 (continued)

Run No.5

F_N = 117.41 lb.

F_N = 53.3 kg

S_N = 7700 psi

s_n = 542 kg/cm^2

Q_A	ψ_R	θ_R	f
240	0.172	13.56 x 10^{-6}	0.042
360	0.183	14.42 "	0.063
480	0.149	11.74 "	0.084
600	0.275	21.67 "	0.105
720	0.218	17.20 "	0.126
840	0.321	25.30 "	0.147
960	0.332	26.20 "	0.168
1080	0.332	26.20 "	0.189
1200	0.470	37.08 "	0.210
1320	0.665	52.50 "	0.231
1440	0.860	67.80 "	0.252
1560	1.250	98.70 "	0.273
1680	- - -	- - - -	0.294

Run No.6

F_N = 64.8 lb.

F_N = 29.4 kg

S_N = 4250 psi

s_n = 299 kg/cm^2

Q_A	ψ_R	θ_R	f
240	- - -	- - -	0.0762
360	0.057	4.50 x 10^{-6}	0.1144
480	0.183	14.42 "	0.1522
600	0.218	17.20 "	0.1905
720	3.630	28.60 "	0.2280
840	- - -	- - -	0.2660

SPECIMEN NO.4

Material: C-1018 Steel

Surface Finish:
2/0 paper

$D_o = 0.2365"$

$D_i = 0.191"$

$L = 1.0078"$

$D_o^2 - D_i^2 = 0.01945$

$D_o^4 - D_i^4 = 0.001798$

$A = \frac{\pi}{4}(D_o^2 - D_i^2) = 0.01526 \text{ in}^2$

$S_N = \frac{F_N}{0.01526}$ psi

$S_T = 1.475 \times Q_A$ psi

$$\psi_c = \frac{77.3 \times L \times Q_A}{(D_o^4 - D_i^4) \times E_s}$$

$\psi_c = 0.00362\ Q_A$ min.arc.

$\psi_c = 0.01144\ \psi_s$ min.arc.

			Run No.1	Run No.2	Run No.3
			F_N = 81.75 lb.	F_N = 116.3 lb.	F_N = 167 lb.
			S_N = 5360 psi	S_N = 7640 psi	S_N = 10940 psi
Q_A	S_T	ψ_c	ψ_o	ψ_o	ψ_o
120	177	0.4345	0.619	0.389	0.447
240	354	0.8695	1.317	0.953	0.825
300	443	1.090	- - -	1.304	- - -
360	532	1.304	- - -	- - -	1.237
420	620	1.520	- - -	- - -	- - -
480	710	1.740	- - -	- - -	1.674
600	886	2.177	- - -	- - -	2.200
720	1065	2.608	- - -	- - -	2.650
840	1243	3.040	- - -	- - -	3.200
900	1330	3.260	- - -	- - -	- - -
960	1420	3.470	- - -	- - -	- - -
1020	1505	3.690	- - -	- - -	- - -
1080	1595	3.910	- - -	- - -	- - -
1140	1680	4.130	- - -	- - -	- - -
1200	1770	4.350	- - -	- - -	- - -
1320	1950	4.775	- - -	- - -	- - -

SPECIMEN NO.4 (continued)

			<u>Run No.4</u>	<u>Run No.5</u>	<u>Run No.6</u>
			F_N = 200.3 lb.	F_N = 200.3 lb.	F_N = 251.2 lb.
			S_N = 13130 psi	S_N = 13130 psi	S_N = 16450 psi
Q_A	S_T	ψ_c	ψ_o	ψ_o	ψ_o
120	177	0.4345	0.526	0.367	0.378
240	354	0.8695	0.814	0.780	0.860
300	443	1.090	0.953	- - -	- - -
360	532	1.304	- - -	1.204	1.225
420	620	1.520	1.306	- - -	- - -
480	710	1.740	- - -	1.660	1.706
600	886	2.177	2.085	2.061	2.130
720	1065	2.608	2.520	2.400	2.590
840	1243	3.040	- - -	2.956	3.050
900	1330	3.260	3.130	- - -	- - -
960	1420	3.470	- - -	3.370	3.525
1020	1505	3.690	3.670	- - -	- - -
1080	1595	3.910	- - -	3.850	4.000
1140	1680	4.130	4.260	- - -	- - -
1200	1770	4.350	- - -	4.275	4.340
1320	1950	4.775	- - -	- - -	4.765

SPECIMEN NO.5

Material: C-1018 Steel

Surface Finish:
4/0 paper

$D_o = 0.2365"$

$D_i = 0.191"$

$L = 1.0078"$

$D_o^2 - D_i^2 = 0.01945$

$D_o^4 - D_i^4 = 0.001798$

$A = 0.01526 \text{ in}^2$

$S_N = \frac{F_N}{0.01526}$ psi

$S_T = 1.475 \times Q_A$ psi

$\psi_c = 0.00362\ Q_A$

$\psi_o = 0.01144\ \psi_s$

			Run No.1	Run No.2
			$F_N = 116.3$ lb.	$F_N = 167$ lb.
			$S_N = 7640$ psi	$S_N = 10940$ psi
Q_A	S_T	ψ_c	ψ_o	ψ_o
120	177	0.4345	0.378	0.446
150	221	0.543	- - -	- - -
240	354	0.8695	0.836	0.940
300	443	1.090	- - -	- - -
360	532	1.304	1.237	1.443
450	664	1.627	- - -	- - -
480	710	1.740	1.750	1.912
600	886	2.177	2.154	2.300
720	1065	2.608	2.610	2.656
750	1105	2.714	- - -	- - -
840	1243	3.040	3.120	3.208
900	1330	3.260	- - -	- - -
960	1420	3.470	3.640	3.676
1050	1550	3.800	- - -	- - -
1080	1595	3.910	- - -	4.420
1200	1770	4.350	- - -	- - -
1320	1950	4.775	- - -	- - -
1350	1990	4.885	- - -	- - -

SPECIMEN NO. 5 (continued)

			Run No.3	Run No.4
			F_N = 200.2 lb.	F_N = 251.2 lb.
			S_N = 14350 psi	S_N = 16450 psi
Q_A	S_T	ψ_c	ψ_o	ψ_o
120	177	0.4345	0.344	- - -
150	221	0.543	- - -	0.493
240	354	0.8695	0.847	- - -
300	443	1.090	- - -	1.040
360	532	1.304	1.340	- - -
450	664	1.627	- - -	1.615
480	710	1.740	1.730	- - -
600	886	2.177	2.185	2.185
720	1065	2.608	2.635	- - -
750	1105	2.714	- - -	2.760
840	1243	3.040	3.080	- - -
900	1330	3.260	- - -	3.332
960	1420	3.470	3.550	- - -
1050	1550	3.800	- - -	3.800
1080	1595	3.910	4.020	- - -
1200	1770	4.350	4.465	4.500
1320	1950	4.775	4.930	- - -
1350	1990	4.885	- - -	5.020

SPECIMEN NO.5

Run No.5

$F_N = 168.16$ lb.

$F_N = 76.4$ kg

$S_N = 11020$ psi

$s_n = 775$ kg/cm^2

Q_A	s_t	ϕ_R	θ_R	f
240	24.9	- - -	- - -	0.0293
360	37.4	- - -	- - -	0.0440
480	49.9	- - -	- - -	0.0587
600	62.3	- - -	- - -	0.0734
720	74.9	- - -	- - -	0.0880
840	87.5	- - -	- - -	0.1026
960	99.9	0.069	5.44×10^{-6}	0.1172
1080	112.0	0.069	5.44 "	0.1320
1200	124.3	0.114	8.98 "	0.1466
1320	137.0	0.151	11.90 "	0.1613
1440	149.4	0.218	17.16 "	0.1760
1560	161.6	0.264	20.80 "	0.1906
1680	174.3	0.447	35.25 "	0.2050
1800	186.6	0.734	57.80 "	0.2200

SPECIMEN NO.5 (continued)

Run No.6

F_N = 117.41 lb.

F_N = 53.3 kg

S_N = 770 psi

s_n = 542 kg/cm^2

Q_A	ψ_R	θ_R	f
240	- - -	- - -	- - -
360	0.034	2.68 x 10^{-6}	0.042
480	0.046	3.63 "	0.063
600	0.218	17.20 "	0.084
720	0.183	14.42 "	0.105
840	0.367	28.90 "	0.126
960	0.458	36.10 "	0.147
1080	0.780	61.50 "	0.168
1200	1.075	84.70 "	0.189
1320	1.396	109.80 "	0.210
1440	1.775	139.80 "	0.231
1560	- - -	- - -	0.252

Run No.7

F_N = 64.78 lb.

F_N = 29.4 kg

S_N = 4250 psi

s_n = 299 kg/cm^2

Q_A	ψ_R	θ_R	f
240	0.183	14.42 x 10^{-6}	0.076
360	0.183	14.42 "	0.114
480	0.412	32.50 "	0.152
600	0.734	57.80 "	0.191
720	0.734	57.80 "	0.228
840	0.768	60.60 "	0.266
960	1.752	138.2 "	0.305

SPECIMEN NO.6

Material: A-4140 Steel

Control Specimen
(without interface)

$D_o = 0.2315"$

$D_i = 0.191"$

$L = 0.9063"$

$D_o^2 - D_i^2 = 0.01711$

$D_o^4 - D_i^4 = 0.001542$

$A = \frac{\pi}{4}(D_o^2 - D_i^2) = 0.01342 \text{ in}^2$

$S_N = \frac{F_N}{A} = \frac{F_N}{0.01342}$ psi

$S_T = 1.121 \times 10^{-2} \frac{D_o}{D_o^4 - D_i^4} \times Q_A$

$S_T = 1.684\, Q_A$ psi

$\psi_c = \frac{77.3 \times L \times Q_A}{(D_o^4 - D_i^4) \times E_s}$

$\psi_c = 0.00382\, Q_A$

$E_s = 11.9 \times 10^6$ psi

$\psi_o = 0.01144\, \psi_s$

Run No.1

$F_N = 251.2$ lb.

$S_N = 18700$ psi

Q_A	S_T	ψ_c	ψ_o
120	202.3	0.458	0.493
240	405	0.916	0.952
360	606.5	1.374	1.410
480	810	1.830	1.845
600	1010	2.290	2.315
720	1213	2.746	2.808
840	1416	3.205	3.323
960	1620	3.660	3.736
1080	1820	4.125	4.160

Run No.2

$F_N = 200.2$ lb.

$S_N = 14900$ psi

Q_A	S_T	ψ_c	ψ_o
150	252.6	0.573	0.596
300	506	1.145	1.065
450	759	1.720	1.592
600	1010	2.290	2.200
750	1263	2.865	2.740
900	1516	3.440	3.320

SPECIMEN NO.7

Material: A-4140 Steel

Surface Finish
As machined (lathe)

$D_o = 0.2315"$

$D_i = 0.191"$

$L = 1.0078"$

$D_o^2 - D_i^2 = 0.01711$

$D_o^4 - D_i^4 = 0.001542$

$A = 0.01342 \text{ in}^2$

$S_N = \frac{F_N}{A} = \frac{F_N}{0.01342}$ psi

$S_T = 1.121 \times 10^{-2} \frac{D_o}{D_o^4 - D_i^4} \times Q_A$

$S_T = 1.684 \times Q_A$ psi

$\psi_c = \frac{77.3 \times L \times Q_A}{(D_o^4 - D_i^4) \times E_s}$

$\psi_c = 0.00425\ Q_A$

$\psi_o = 0.01144\ \psi_s$

			Run No.1	Run No.2
			F_N = 116.3 lb.	F_N = 167 lb.
			S_N = 8700 psi	S_N = 12470 psi
Q_A	S_T	ψ_c	ψ_o	ψ_o
120	202	0.510	0.367	0.436
240	404	1.020	0.849	0.895
300	505	1.274	- - -	- - -
360	606	1.530	1.410	1.490
420	706	1.785	- - -	- - -
480	808	2.040	1.937	2.140
600	1010	2.546	2.510	2.720
720	1212	3.057	- - -	3.370
900	1515	3.820	- - -	- - -
1020	1718	4.340	- - -	- - -

SPECIMEN NO.7 (continued)

			Run No.3	Run No.4
			F_N = 200.2 lb.	F_N = 251.2 lb.
			S_N = 14950 psi	S_N = 18740 psi
Q_A	S_T	ψ_c	ψ_o	ψ_o
120	202	0.510	0.435	0.482
240	404	1.020	- - -	- - -
300	505	1.274	1.237	1.363
360	606	1.530	- - -	- - -
420	706	1.785	1.753	1.850
480	808	2.040	- - -	- - -
600	1010	2.546	2.450	2.418
720	1212	3.057	3.092	- - -
900	1515	3.820	4.075	3.940
1020	1718	4.340	- - -	4.550

SPECIMEN NO. 8

Material: A-4140 Steel

Surface Finish:
No.1 emery paper

$D_o = 0.2315''$

$D_i = 0.191''$

$L = 1.0078''$

$D_o^2 - D_i^2 = 0.0177$

$D_o^4 - D_i^4 = 0.001542$

$A = 0.01342 \text{ in}^2$

$S_N = \frac{F_N}{0.01342}$ psi

$S_T = 1.684 \times Q_A$ psi

$\psi_c = 0.00425\ Q_A$

$\psi_o = 0.01144\ \psi_s$

			Run No.1	Run No.2
			$F_N = 116.3$ lb.	$F_N = 167$ lb.
			$S_N = 8700$ psi	$S_N = 12470$ psi
Q_A	S_T	ψ_c	ψ_o	ψ_o
150	252.5	0.637	0.618	0.676
300	505	1.274	1.270	1.330
450	758	1.910	1.890	1.923
600	1010	2.546	2.576	2.550
750	1262	3.185	3.260	3.170
900	1515	3.820	- - -	- - -
1050	1770	4.460	- - -	- - -

SPECIMEN NO. 8 (continued)

			Run No.3	Run No.4
			F_N = 200.2 lb.	F_N = 251.2 lb.
			S_N = 14950 psi	S_N = 18740 psi
Q_A	S_T	ψ_c	ψ_o	ψ_o
150	252.5	0.637	0.551	0.585
300	505	1.274	1.250	1.144
450	758	1.910	2.042	1.846
600	1010	2.546	2.520	2.510
750	1262	3.185	3.164	3.290
900	1515	3.820	3.890	3.900
1050	1770	4.460	- - -	4.510

SPECIMEN NO.8

Run No.7

F_N = 201.29 lb.

F_N = 91.3 kg

S_N = 15000 psi

s_n = 1053 kg/cm^2

Q_A	s_t	ψ_R	θ_R	f
120	14.2	- - -	- - -	0.0124
240	28.4	- - -	- - -	0.0299
360	42.6	- - -	- - -	0.0373
480	56.8	- - -	- - -	0.0597
600	71.0	- - -	- - -	0.0621
720	85.3	- - -	- - -	0.0746
840	99.5	- - -	- - -	0.0870
960	113.5	0.011	0.86×10^{-6}	0.0995
1080	127.7	0.046	3.59 "	0.1119
1200	142.0	0.057	4.45 "	0.1242
1320	156.2	0.092	7.18 "	0.1367
1440	170.4	0.092	7.18 "	0.1490
1560	184.5	0.138	10.75 "	0.1615
1680	198.7	0.252	19.65 "	0.1740
1800	213.0	0.412	32.15 "	0.1864
1920	227.0	0.803	62.60 "	0.1990
2040	241.0	0.859	67.00 "	0.2113
2160	256.0	1.512	118.0 "	0.2240
2280	270.0	2.660	207.4 "	0.2360
2400	284.0	- - -	- - - -	0.2485

SPECIMEN NO.8 (continued)

<u>Run No.8</u>

F_N = 168.16 lb.

F_N = 76.4 kg

S_N = 12540 psi

s_n = 883 kg/cm^2

Q_A	ϕ_R	θ_R	f
120	- - -	- - -	0.0149
240	- - -	- - -	0.0298
360	- - -	- - -	0.0446
480	- - -	- - -	0.0595
600	0.114	8.89 x 10^{-6}	0.0745
720	0.401	31.25 "	0.0894
840	0.481	37.50 "	0.1042
960	0.905	70.60 "	0.1190
1080	0.974	76.00 "	0.1340
1200	0.940	73.35 "	0.1487
1320	1.132	88.30 "	0.1636
1440	1.317	102.6 "	0.1786
1560	1.545	120.5 "	0.1935
1680	2.395	186.6 "	0.2085
1800	2.575	200.8 "	0.2232
1920	2.740	214.0 "	0.2380
2040	3.045	237.5 "	0.2530
2160	5.440	424.0 "	0.2680
2280	- - -	- - - -	0.2830

<u>Run No.9</u>

F_N = 117.41 lb.

F_N = 53.3 kg

S_N = 8760 psi

s_n = 616 kg/cm^2

Q_A	ϕ_R	θ_R	f
120	- - -	- - -	0.0213
240	- - -	- - -	0.0426
360	0.138	10.75 x 10^{-6}	0.0640
480	0.252	19.64 "	0.0852
600	0.653	50.90 "	0.1066
720	1.042	81.30 "	0.1280
840	1.820	141.8 "	0.1492
960	3.240	252.5 "	0.1705
1080	- - -	- - -	0.1916

SPECIMEN NO.8 (continued)

Run No.10

F_N = 64.78 lb.

F_N = 29.4 kg

S_N = 4835 psi

s_n = 340 kg/cm^2

Q_A	ψ_R	θ_R	f
120	- - -	- - -	0.0386
240	0.172	13.40 x 10^{-6}	0.0773
360	0.836	65.20 "	0.1160
480	8.850	690.0 "	0.1545
600	- - -	- - -	0.1930

SPECIMEN NO.9

Material: A-4140 Steel

Surface Finish:
2/0 paper

$D_o = 0.2315"$

$D_i = 0.191"$

$L = 1.0078"$

$D_o^2 - D_i^2 = 0.01711$

$D_o^4 - D_i^4 = 0.001542$

$A = 0.01342 \text{ in}^2$

$S_N = \frac{F_N}{0.01342}$

$S_T = 1.684 \times Q_A$ psi

$\psi_c = 0.00425\ Q_A$

$\psi_o = 0.01144\ \psi_s$

			Run No.1	Run No.2
			F_N = 116.3 lb.	F_N = 167 lb.
			S_N = 8700 psi	S_N = 12470 psi
Q_A	S_T	ψ_c	ψ_o	ψ_o
150	252.5	0.637	0.607	0.425
300	505	1.274	1.306	1.240
450	758	1.910	2.027	1.800
600	1010	2.546	2.635	2.363
750	1262	3.185	- - -	2.910
900	1515	3.820	- - -	3.650
1050	1770	4.460	- - -	4.410

SPECIMEN NO. 9 (continued)

			Run No.3	Run No.4
			F_N = 200.2 lb.	F_N = 251.2 lb.
			S_N = 14950 psi	S_N = 18740 psi
Q_A	S_T	ϕ_c	ϕ_o	ϕ_o
150	252.5	0.637	0.551	0.768
300	505	1.274	1.390	1.240
450	758	1.910	1.893	2.009
600	1010	2.546	2.480	2.560
750	1262	3.185	3.020	3.155
900	1515	3.820	3.760	- - -
1050	1770	4.460	- - -	- - -

SPECIMEN NO. 10.

Material: 2S Aluminum

Control Specimen:
(without interface)

$D_o = 0.2365"$

$D_i = 0.191"$

$L = 0.9844"$

$D_o^2 - D_i^2 = 0.01945$

$D_o^4 - D_i^4 = 0.001798$

$A = \frac{\pi}{4}(D_o^2 - D_i^2) = 0.01526$

$S_N = \frac{F_N}{A} = \frac{F_N}{0.01526}$ psi

$S_T = 1.121 \times 10^{-2} \frac{D_o}{D_o^4 - D_i^4} \times Q_A$

$S_T = 1.475 \times Q_A$ psi

$\psi_c = \frac{77.3 \times L \times Q_A}{(D_o^4 - D_i^4) \times E_s}$

$E_s = 3.7 \times 10^6$ psi

$\psi_c = 0.01145 \times Q_A$

$\psi_o = 0.01144\ \psi_s$

			Run No.1		Run No.2	
			F_N = 116.3 lb.		F_N = 167 lb.	
			S_N = 7640 psi		S_N = 10940 psi	
Q_A	S_T	ψ_c	ψ_o	ψ_R	ψ_o	ψ_R
30	44.3	0.343	0.321	- - -	0.252	0.069
60	88.5	0.685	0.687	- - -	0.573	0.057
90	132.8	1.028	1.100	0.218	1.144	0.046
120	177	1.370	1.362	0.218	- - -	- - -
150	221.5	1.714	1.740	0.344	1.545	0.241
180	265.5	2.056	2.072	0.298	- - -	- - -
210	310	2.400	2.415	0.367	2.530	0.344
240	354	2.740	2.865	0.447	- - -	- - -
270	398	3.085	3.108	0.458	3.115	0.355
300	443	3.430	3.390	0.482	- - -	- - -
330	487	3.770	3.700	0.402	3.850	0.401
360	531	4.110	4.125	0.505	- - -	- - -
420	620	4.810	- - -	- - -	4.500	0.607

ψ_R = Residual Twist. Sight edges on indicator arms failed to return to zero index on removal of torque. ψ_R is residual twist remaining in specimen after removal of torque.

SPECIMEN NO.11

Material: Copper

Control Specimen:
(without interface)

$D_o = 0.2368"$

$D_i = 0.191"$

$L = 1.01563"$

$D_o^2 - D_i^2 = 0.01959$

$D_o^4 - D_i^4 = 0.001814$

$A = \frac{\pi}{4}(D_o^2 - D_i^2) = 0.01537$

$S_N = \frac{F_N}{0.01537}$ psi

$S_T = 1.121 \times 10^{-2} \frac{D_o}{D_o^4 - D_i^4} Q_A$

$S_T = 1.464 \times Q_A$ psi

$\psi_c = \frac{77.3 \times L \times Q_A}{(D_o^4 - D_i^4) \times E_s}$

$E_s = 6.0 \times 10^6$ psi

$\psi_c = 0.00722\ Q_A$

$\psi_o = 0.01144\ \psi_s$

			Run No.1		Run No.2		Run No.3	
			F_N = 200 lb.		F_N = 250 lb.		F_N = 200 lb.	
			S_N = 13020 psi		S_N = 16270 psi		S_N = 13020 psi	
Q_A	S_T	ψ_c	ψ_o	ψ_R	Q_A	ψ_R	Q_A	ψ_R
30	44	0.216	0.332	- - -	30	- - -	120	0.050
60	88	0.434	0.550	0.040	60	0.046	240	0.050
90	132	0.650	0.780	0.160	90	- - -	360	0.092
120	176	0.866	0.974	0.040	120	0.080	480	0.090
150	220	1.082	1.052	0.060	150	- - -	600	0.103
180	264	1.300	1.340	0.161	180	0.050	720	0.217
210	308	1.515	1.580	0.320	210	- - -	840	0.320
240	352	1.730	1.763	0.252	240	0.069	960	0.310
270	396	1.950	1.980	0.321	270	- - -	1080	0.286
300	440	2.165	2.185	0.424	300	0.172	1200	0.458
330	484	2.380	2.415	0.401	330	- - -	1320	0.573
360	527	2.596	2.610	0.424	360	0.092	1440	0.652
390	572	2.813	2.840	0.504	390	- - -	1560	0.812
420	615	3.030	2.986	0.550	420	0.252	1680	0.995
450	660	3.245	3.220	0.642	450	- - -	1800	0.965

SPECIMEN NO.12

Run No.1

F_N = 31.86 lb.

F_N = 14.4 kg

S_N = 2375 psi

s_n = 167 kg/cm^2

Q_A	s_t	ψ_R	θ_R	f
120	14.2	- - -	- - -	0.0785
240	28.4	0.046	3.59×10^{-6}	0.1570
360	42.6	0.149	11.60 "	0.2358
480	56.8	0.825	64.40 "	0.3140
600	71.0	- - -	- - -	0.3930
720	85.3			
840	99.5			
960	113.5			
1080	127.7			
1200	142.0			
1320	156.2			
1440	170.4			
1560	184.5			
1680	198.7			
1800	213.0			

Material: A-4140 Steel
Surface Finish: 4/0 paper

SPECIMEN NO.12 (continued)

<u>Run No.2</u>

F_N = 53.8 lb.

F_N = 24.2 kg

S_N = 3980 psi

s_n = 280 kg/cm^2

Q_A	ψ_R	θ_R	f
120	0.114	8.89×10^{-6}	0.0469
240	0.184	14.34 "	0.0938
360	0.321	25.05 "	0.1406
480	0.860	67.15 "	0.1875
600	2.580	201.3 "	0.2344
720	3.085	240.6 "	0.2814
840	- - -	- - - -	0.3280

<u>Run No.3</u>

F_N = 117.41 lb.

F_N = 53.3 kg

S_N = 8760 psi

s_n = 616 kg/cm^2

Q_A	ψ_R	θ_R	f
120	- - -	- - - -	0.0213
240	- - -	- - - -	0.0426
360	0.011	0.86×10^{-6}	0.0640
480	0.046	3.59 "	0.0862
600	0.184	14.34 "	0.1066
720	0.665	51.90 "	0.1280
840	0.688	53.65 "	0.1492
960	0.872	68.00 "	0.1705
1080	1.053	82.20 "	0.1916
1200	1.144	89.30 "	0.2130
1320	1.707	133.0 "	0.2340
1440	2.096	163.3 "	0.2555
1560	3.060	238.6 "	0.2768
1680	7.550	589.0 "	0.2980
1800	- - -	- - - -	0.3190

SPECIMEN NO.13

Material: Copper

Surface Finish: No.1 Paper

$D_o = 0.2359"$

$D_i = 0.191"$

$L = 1.000"$

$S_N = \dfrac{F_N}{0.01505}$ psi

$S_T = 1.495 \times Q_A$ psi

$\psi_c = 0.00708 \times Q_A$

$E_s = 6.0 \times 10^6$ psi

$\psi_o = 0.01144\ \psi_s$

			Run No.2	Run No.3		Run No.1	
			F_N = 201.3 lb.	F_N = 251 lb.		F_N = 168.2 lb.	
			S_N = 13360 psi	S_N = 16700 psi		S_N = 11170 psi	
Q_A	S_T	ψ_c	ψ_o	Q_A	ψ_o	Q_A	ψ_o
60	90	0.424	0.298	60	- - -	60	0.515
120	180	0.850	0.780	120	0.619	120	0.858
180	269	1.273	1.270	180	- - -	180	1.328
240	359	1.700	1.730	240	1.087	240	1.730
300	449	2.125	2.180	300	- - -	300	- - -
360	539	2.550	2.670	360	2.520		
420	629	2.970	2.920	420	- - -		
480	719	3.400	3.380	480	3.345		
540	808	3.820	3.840	540	- - -		
600	900	4.250	4.350	600	4.330		
660	987	4.670	4.750	660	- - -		
720	1076	5.100	5.210	720	5.30		
780	1167	5.520	5.710	780	- - -		
840	1257	5.950	6.215	840	6.260		
960	1436	6.800	- - -	960	- - -		

SPECIMEN NO.13

Run No.1

$F_N = 168.16$ lb.

$F_N = 76.4$ kg

$S_N = 11170$ psi

$s_n = 785$ kg/cm^2

Q_A	s_t	ψ_R	θ_R	f
60	6.3	- - -	- - -	0.0074
120	12.6	0.080	6.34×10^{-6}	0.0147
180	18.9	0.103	8.15 "	0.0220
240	25.2	0.114	9.04 "	0.0294
300	31.5	- - -	- - -	0.0367

Run No.5

$F_N = 117.41$ lb.

$F_N = 53.3$ kg

$S_N = 7800$ psi

$s_n = 548$ kg/cm^2

Q_A	ψ_R	θ_R	f
120	- - -	- - -	0.0211
240	0.023	1.81×10^{-6}	0.0421
360	0.126	9.95 "	0.0632
480	0.149	11.76 "	0.0842
600	0.183	14.45 "	0.1052
720	0.298	23.50 "	0.1264
840	- - -	- - -	0.1475

SPECIMEN NO.13 (continued)

Run No.2

$F_N = 201.29$ lb.

$F_N = 91.3$ kg

$S_N = 13360$ psi

$s_n = 939$ kg/cm^2

Q_A	ψ_R	θ_R	f
60	- - -	- - -	0.00615
120	- - -	- - -	0.0123
180	0.149	11.76×10^{-6}	0.0184
240	0.069	5.42 "	0.0246
300	0.149	11.76 "	0.0308
360	0.137	10.85 "	0.0369
420	0.309	24.40 "	0.0430
480	0.275	21.72 "	0.0491
540	0.286	22.60 "	0.0553
600	0.367	29.00 "	0.0615
660	0.458	36.20 "	0.0676
720	0.435	34.40 "	0.0737
780	0.435	34.40 "	0.0800
840	0.446	35.22 "	0.0860
960	- - -	- - -	- - -

Run No.3

$F_N = 251$ lb.

$F_N = 114.0$ kg

$S_N = 16700$ psi

$s_n = 1172$ kg/cm^2

Q_A	ψ_R	θ_R	f
120	- - -	- - -	0.0098
240	- - -	- - -	0.0197
360	0.114	9.00×10^{-6}	0.0295
480	0.046	3.62 "	0.0394
600	0.183	14.45 "	0.0493
720	0.435	34.40 "	0.0640
840	0.618	48.80 "	0.0690
960	- - -	- - -	- - -

SPECIMEN NO.13 (continued)

<u>Run No.6</u>

F_N = 168.16 lb.

F_N = 76.4 kg

S_N = 11170 psi

s_n = 785 kg/cm^2

Q_A	ϕ_R	θ_R	f
120	- - -	- - -	0.0147
240	0.092	7.24 x 10^{-6}	0.0294
360	0.080	6.33 "	0.0441
480	0.057	4.52 "	0.0588
600	0.092	7.24 "	0.0735
720	0.195	15.35 "	0.0882
840	0.481	38.00 "	0.1028
960	0.722	57.00 "	0.1175
1080	0.929	73.30 "	0.1320
1200	- - -	- - -	0.1470

<u>Run No.7</u>

F_N = 201.29 lb.

F_N = 91.3 kg

S_N = 13360 psi

s_n = 939 kg/cm^2

Q_A	ϕ_R	θ_R	f
120	- - -	- - -	0.0123
240	0.011	0.90 x 10^{-6}	0.0246
360	0.092	7.24 "	0.0369
480	0.172	13.56 "	0.0491
600	0.149	11.76 "	0.0615
720	0.218	17.15 "	0.0737
840	0.321	25.30 "	0.0860
960	0.458	36.10 "	0.0983
1080	0.493	38.90 "	0.1105
1200	0.550	43.30 "	0.1228
1320	0.825	65.10 "	0.1350
1440	2.120	167.0 "	0.1473
1560	2.820	222.4 "	0.1596
1680	- - -	- - -	0.1720

SPECIMEN NO.14

Material: Copper

Surface Finish: 2/o Paper

$D_o = 0.2359"$

$D_i = 0.191"$

$L = 1.000"$

$S_N = \frac{F_N}{0.01505}$ psi

$S_T = 1.495 \times Q_A$ psi

$\psi_c = 0.00708 \times Q_A$

$E_s = 6.0 \times 10^6$ psi

$\psi_o = 0.01144\ \psi_s$

			Run No.1	Run No.2	
			F_N = 64.8 lb.	F_N = 117.4 lb.	
			S_N = 4300 psi	S_N = 7800 psi	
Q_A	S_T	ψ_c	ψ_o	Q_A	ψ_o
120	180	0.850	0.837	120	0.756
240	359	1.700	1.740	240	1.560
360	539	2.550	2.715	360	2.510
480	719	3.400	3.910	480	3.480
600	900	4.250	5.320	600	4.500
720	1076	5.100		720	5.730
840	1257	5.950		840	5.970
960	1436	6.800			
1080	1614	7.650			

SPECIMEN NO.14 (continued)

			Run No.3	Run No.4		Run No.5	
			F_N = 168.2 lb.	F_N = 201.3 lb.		F_N = 231.68 lb.	
			S_N = 11170 psi	S_N = 13360 psi		S_N = 15400 psi	
Q_A	S_T	ψ_c	ψ_o	Q_A	ψ_o	Q_A	ψ_o
120	180	0.850	0.733	120	0.780	120	0.882
240	359	1.700	1.525	240	1.696	240	1.294
360	539	2.550	2.350	360	2.570	360	2.610
480	719	3.400	3.080	480	3.475	480	3.540
600	900	4.250	4.040	600	4.290	600	4.380
720	1076	5.100	4.940	720	5.360	720	5.333
840	1257	5.950	5.740	840	6.310	840	6.230
960	1436	6.800	6.720	960	7.230	960	7.250
1080	1614	7.650	- - - - - - - - - - off scope - - - - - - - - -				

SPECIMEN NO.14

Run No.1

F_N = 64.78 lb.

F_N = 29.4 kg

S_N = 4300 psi

s_n = 302 kg/cm^2

Q_A	s_t	ψ_R	θ_R	f
120	12.6	- - -	- - -	0.0382
240	25.2	0.183	14.46×10^{-6}	0.0763
360	37.8	0.298	23.50 "	0.1144
480	50.5	0.505	39.88 "	0.1526
600	63.1	3.310	261.5 "	0.1910
720	75.6	- - -	- - -	0.2290
840	88.4			
960	101.0			
1080	113.6			
1200	126.1			
1320	138.9			
1440	151.2			
1560	164.0			
1680	176.6			
1800	189.0			
1920	201.5			
2040	214.2			
2160	226.8			
2280	239.5			
2400	252.0			
2520	264.0			

SPECIMEN NO.14 (continued)

Run No.2

$F_N = 117.41$ lb.

$F_N = 53.3$ kg

$S_N = 7800$ psi

$s_n = 548$ kg/cm^2

Q_A	ψ_R	θ_R	f
120	- - -	- - -	0.0211
240	0.057	4.50×10^{-6}	0.0421
360	0.184	14.50 "	0.0632
480	0.378	29.80 "	0.0842
600	0.653	51.50 "	0.1052
720	1.032	81.50 "	0.1264
840	1.420	112.0 "	0.1475
960	3.370	294.3 "	0.1683
1080	10.000	790.0 "	0.1893
1200	- - -	- - -	0.2100

Run No.3

$F_N = 168.16$ lb.

$F_N = 764$ kg

$S_N = 11170$ psi

$s_n = 785$ kg/cm^2

Q_A	ψ_R	θ_R	f
120	- - -	- - -	0.0147
240	- - -	- - -	0.0294
360	- - -	- - -	0.0441
480	- - -	- - -	0.0588
600	0.057	4.50×10^{-6}	0.0735
720	0.149	11.76 "	0.0882
840	0.172	13.56 "	0.1028
960	0.241	19.00 "	0.1175
1080	0.355	28.00 "	0.1320
1200	0.390	30.78 "	0.1470
1320	0.424	33.24 "	0.1616
1440	0.734	57.90 "	0.1764
1560	- - -	- - -	0.1910

SPECIMEN NO.14 (continued)

Run No.4

F_N = 201.29 lb.

F_N = 91.3 kg

S_N = 13360 psi

s_n = 939 kg/cm^2

Q_A	ψ_R	θ_R	f
120	- - -	- - -	0.0123
240	0.184	14.50×10^{-6}	0.0246
360	0.264	20.40 "	0.0369
480	0.435	34.40 "	0.0491
600	0.550	43.40 "	0.0615
720	0.723	57.00 "	0.0737
840	0.769	60.60 "	0.0860
960	0.906	71.50 "	0.0983
1080	1.007	79.60 "	0.1105
1200	1.087	85.80 "	0.1228
1320	1.363	107.5 "	0.1350
1440	1.546	121.8 "	0.1473
1560	2.040	161.0 "	0.1596
1680	2.222	175.0 "	0.1720
1800	2.500	197.0 "	0.1845
1920	2.960	233.5 "	0.1966
2040	- - -	- - -	0.2090

Run No.5

F_N = 231.68 lb.

F_N = 105.0 kg

S_N = 15400 psi

s_n = 1080 kg/cm^2

Q_A	ψ_R	θ_R	f
120	- - -	- - -	0.0107
240	0.160	12.62×10^{-6}	0.0214
360	0.321	25.30 "	0.0320
480	0.424	33.42 "	0.0427
600	0.493	38.90 "	0.0535
720	0.619	48.80 "	0.0640
840	0.756	59.65 "	0.0748
960	0.974	76.80 "	0.0855
1080	0.962	75.90 "	0.0962
1200	1.100	87.00 "	0.1069
1320	1.340	105.6 "	0.1175
1440	1.350	106.4 "	0.1280
1560	1.593	125.6 "	0.1390
1680	1.775	140.0 "	0.1495
1800	1.890	149.0 "	0.1600
1920	2.060	162.5 "	0.1708
2040	2.325	183.5 "	0.1815
2160	2.690	212.0 "	0.1920
2280	2.990	236.0 "	0.2030
2400	3.350	264.0 "	0.2135
2520	- - -		[illegible]

SPECIMEN NO.15

Run No.1

F_N = 201.29 lb.

F_N = 91.3 kg

S_N = 13360 psi

s_n = 939 kg/cm^2

Q_A	s_t	ψ_R	θ_R		f
120	12.6	- - -	- - -		0.0123
240	25.2	- - -	- - -		0.0246
360	37.8	0.069	5.42	x 10^{-6}	0.0368
480	50.5	- - -	- - -		0.0491
600	63.1	0.161	12.90	"	0.0614
720	75.6	0.011	0.90	"	0.0736
840	88.4	0.057	4.50	"	0.0860
960	101.0	0.034	2.68	"	0.0982
1080	113.6	0.069	5.42	"	0.1104
1200	126.1	0.126	9.95	"	0.1226
1320	138.9	0.252	19.90	"	0.1350
1440	151.2	0.355	28.00	"	0.1472
1560	164.0	0.458	36.20	"	0.1595
1680	176.6	0.653	51.55	"	0.1716
1800	189.0	0.734	57.90	"	0.1840

Material: Copper

Surface Finish: 4/0 paper

SPECIMEN NO.15 (continued)

Run No.2

F_N = 168.16 lb.

F_N = 76.4 kg

S_N = 11170 psi

s_n = 785 kg/cm^2

Q_A	ϕ_R	θ_R	f
120	- - -	- - -	0.0147
240	0.011	0.90 x 10^{-6}	0.0294
360	0.023	1.81 "	0.0441
480	- - -	- - -	0.0588
600	0.011	0.90 "	0.0735
720	0.034	2.68 "	0.0882
840	0.034	2.68 "	0.1028
960	0.069	5.42 "	0.1175
1080	0.103	8.15 "	0.1320
1200	0.194	15.30 "	0.1470
1320	0.218	17.20 "	0.1615
1440	0.229	18.09 "	0.1762
1560	0.309	24.40 "	0.1910
1680	0.619	48.85 "	0.2060
1800	- - -	- - -	0.2203

Run No.3

F_N = 117.41 lb.

F_N = 53.3 kg

S_N = 7800 psi

s_n = 548 kg/cm^2

Q_A	ϕ_R	θ_R	f
120	- - -	- - -	0.0211
240	- - -	- - -	0.0421
360	- - -	- - -	0.0632
480	0.023	1.81 x 10^{-6}	0.0842
600	0.126	9.95 "	0.1052
720	0.149	11.76 "	0.1265
840	0.229	18.09 "	0.1475
960	- - -	- - -	0.1686

SPECIMEN NO.15 (continued)

Run No.4

F_N = 168.16 lb.

F_N = 76.4 kg

S_N = 11170 psi

s_n = 785 kg/cm^2

C_A	ϕ_R	θ_R	f
120	- - -	- - -	0.0147
240	0.103	8.15 x 10^{-6}	0.0294
360	0.149	11.76 "	0.0441
480	0.195	15.40 "	0.0588
600	0.344	27.15 "	0.0735
720	0.413	32.60 "	0.0882
840	0.768	60.65 "	0.1028
960	1.134	89.50 "	0.1175
1080	- - -	- - -	0.1320

SPECIMEN NO.15 (continued)

<u>Run No.5</u>

F_N = 117.41 lb.

F_N = 53.3 kg

S_N = 7800 psi

s_n = 548 kg/cm^2

Q_A	ϕ_R	θ_R	f
120	0.080	6.34 x 10^{-6}	0.0211
240	0.321	25.30 "	0.0421
360	0.596	47.10 "	0.0632
480	1.396	110.0 "	0.0842
600	- - -	- - -	0.1052

<u>Run No.7</u>

F_N = 64.78 lb.

F_N = 29.4 kg

S_N = 4300 psi

s_n = 302 kg/cm^2

Q_A	ϕ_R	θ_R	f
120	0.092	7.24 x 10^{-6}	0.0382
240	0.241	19.04 "	0.0763
360	0.940	74.20 "	0.1144
480	8.50	671.0 "	0.1526

SPECIMEN NO.16

Material: 2S Aluminum

Surface Finish: No. 1 Paper

Run No.1

$F_N = 764$ kg

$s_n = 772$ kg/cm^2

C_A	s_t	ψ_R	θ_R	f	ψ_o
12	1.24	0.458	30.10 x 10^{-6}	0.0015	0.183
30	3.11	0.138	10.87 "	0.0037	0.413
60	6.23	0.138	10.87 "	0.0073	0.836
90	9.33	0.275	21.65 "	0.0119	1.064
120	12.43	0.321	25.30 "	0.0147	1.443
150	15.55	0.424	33.40 "	0.0183	1.866
180	18.65	0.596	47.00 "	0.0220	2.085
210	21.80	0.699	55.10 "	0.0256	2.462
240	24.90	0.630	49.65 "	0.0293	2.820
270	28.00	0.722	57.00 "	0.0330	3.130
300	31.15	0.756	59.60 "	0.0367	3.450
330	34.25	0.814	64.15 "	0.0403	3.930
360	37.35	0.836	65.95 "	0.0440	4.220
390	40.50	0.956	75.40 "	0.0476	4.530
420	43.65	0.985	77.60 "	0.0514	4.810

SPECIMEN NO.16 (continued)

Material: 2S Aluminum

Surface Finish: No. 1 Paper

Run No.2

$F_N = 91.3$ kg

$s_n = 928$ kg/cm^2

Q_A	s_t	ψ_R	θ_R	f	ψ_o
12	1.24	- - -	- - -	0.0012	- - -
30	3.11	- - -	- - -	0.0031	0.378
60	6.23	- - -	- - -	0.0061	0.596
90	9.33	0.114	9.00×10^{-6}	0.0092	1.030
120	12.43	0.138	10.87 "	0.0123	1.283
150	15.55	0.183	14.42 "	0.0153	1.640
180	18.65	0.138	10.87 "	0.0184	1.970
210	21.80	0.206	16.24 "	0.0214	2.220
240	24.90	0.252	19.86 "	0.0245	2.625
270	28.00	0.283	22.30 "	0.0276	2.980
300	31.15	0.367	28.90 "	0.0307	3.240
330	34.25				

APPENDIX C

SAMPLE CALCULATIONS

SAMPLE CALCULATIONS

Calculated angle of twist of specimen without interface = ψ_o

$$\psi = \frac{T_A L}{E_s J} \text{ radians} \tag{1}$$

$$J = \frac{\pi(D_o^4 - D_i^4)}{32} \tag{2}$$

$$\psi_c = \frac{180}{\pi} \times 60 \times \psi \tag{3}$$

$$T_A = 2.205 \times 10^{-3}\ Q_A \tag{4}$$

Substituting (2), (3), and (4) in (1) gives

$$\psi_c = \frac{77.3 \times Q_A \times L}{(D_o^4 - D_i^4) \times E_s} \text{ Min.arc.} \tag{5}$$

Maximum tangential stress = S_T.

$$S_T = \frac{16\ T_A D_o}{\pi(D_o^4 - D_i^4)} \tag{6}$$

Substituting (4) in (6) gives

$$S_T = 1.121 \times 10^{-2} \frac{D_o}{D_o^4 - D_i^4} \times Q \text{ psi} \tag{7}$$

Observed angle of twist of specimen = ψ_o.

With 10 x objective calibration with micrometer stage shows one drum unit of optical micrometer represents 0.00004101" on indicator.

ψ_s drum units = 0.00004101 x ψ_s inches on indicator.

$$\psi_o = \frac{\psi_s}{R} \text{ radians} \times \frac{180}{\pi} \frac{\text{degrees}}{\text{radians}} \times 60 \frac{\text{min.}}{\text{degree}} .$$

$$\psi_o = \frac{\psi_s(0.00004101)}{12.33} \times 3440$$

$$\psi_o = 0.01144\ \psi_s \tag{8}$$

Coefficient of friction = f.

$$T = \frac{1}{3} f \times F_N \frac{(D_o^3 - D_i^3)}{(D_o^2 - D_i^2)} \tag{9}$$

T = torque of friction about axis of shaft.

Substituting (4) in (9) gives

$$f = \frac{Q_A}{F_N} \times 6.615 \times 10^{-3} \times \frac{(D_o^2 - D_i^2)}{(D_o^3 - D_i^3)}$$

APPENDIX D

SUPPLEMENTARY DISCUSSION

SUPPLEMENTARY DISCUSSION

In the discussion of results, it was suggested that yield effects in the metal of the interfaces contributed to the small initial deformations. An examination of one of the test runs for this effect is therefore in order.

Referring to Figure XIV, let us examine the stress situation in the C-1018 steel specimen for one of the initial deformations.

The maximum combined shearing stress in a cylinder loaded in this manner is given as follows: $(S_s)_{max} = \sqrt{\frac{1}{4}(S_N)^2 + (S_T)^2}$

For S_N and S_T values, respectively, of 13200 and 1420 psi, the $(S_s)_{max}$ is equal to 6750 psi.

Maximum shear theory states that yielding will occur when $(S_s)_{max}$ equals the maximum shearing stress at yield point obtained from a tension test. The maximum shearing stress at yield point is one half yield stress for a tensile specimen.

Ryerson Steel Specifications for C-1018 steel give a yield value of 48000 psi.

Therefore maximum shear stress at yield point equals 24000 psi. The calculated stress for the specimen is well below this value; therefore yield will not occur in the bulk metal.

The normal stress value used in computing the combined shear stress was determined by using the cross-section area of the specimen, which at the interface is the apparent area of

contact. The real area of contact is less than this value due to the asperities in the metal of the interface.

Bowden and Tabor[4] give the following relation for real area of contact:

$$A_r = \frac{F}{P_m}$$

P_m = mean pressure over area of contact

$P_m = C \times S_{yield}$

C has a value of 3 for material and surface finish used here.

P_m = 144000 psi for this case.

A_{real} = 0.0014 in^2, which is considerably less than the apparent value of 0.0153 in^2.

This would give a maximum combined shear stress in the contact surface considerably in excess of that required to initiate yield in the metal.

It is thus apparent that the stresses at the contact surface are more than sufficient to insure yield in the asperities in the metal interface. For very small values of deformation, the yield effects may thus be the sole contributing factor.

Now consider the stress values for the specimen under conditions encountered just prior to the advent of free sliding to S_N and S_T of 13200 and 4960 psi. The $(S_s)_{max}$ value is 8260 psi. Thus it is apparent that yield in the bulk material of the specimen does not occur, and all but the smallest displacements are due to slip between the two contact surfaces.

APPENDIX E

ORIGINAL DATA

SPECIMEN NO.1

Material: C-1018 Steel

Control Specimen (without interface)

	<u>Run No.1</u>	<u>Run No.2</u>	<u>Run No.3</u>	<u>Run No.4</u>	<u>Run No.5</u>
	F_N = 150 lb.	F_N = 190.7 lb.	F_N = 225.4 lb.	F_N = 74.9 lb.	F_N = 111.9 lb.
Q_A	ψ_s	ψ_s	ψ_s	ψ_s	ψ_s
150	44	44	49	40	45
300	92	89	96	101	97
450	139	143	146	- -	144
600	189	197	193		196
750	239	243	- -		- -
900	285	296	292		
1050	332	- -	342		
1200	- -		391		
1350			- -		

SPECIMEN NO.2

Material: C-1018 Steel

Surface Finish:
As machined (lathe)

Run No.1

F_N = 251.2 lb.

Q_A	ψ_s
120	31
240	60
360	103
480	145
600	187
720	227
840	266
900	282
960	300
1080	342
1200	384
1320	433
1440	478

SPECIMEN NO.3

Material: C-1018 Steel

Surface Finish: No.1 Paper

Run No.1		Run No.2		Run No.3			
F_N = 200.3 lb.		F_N = 251.2 lb.		F_N = 201.29 lb.			
Q_A	ψ_s	Q_A	ψ_s	Q_A	ψ_{sR}	Q_A	ψ_{sR}
120	38	120	36	120	--	1800	30
240	80	240	74	240	--	1920	40
360	123	360	110	360	--	2040	50
480	168	480	144	480	--	2160	56
600	203	600	183.5	600	--	2280	70
720	241	720	226	720	5	2400	82
840	- -	840	268	840	7	2520	97
900		900	- -	900	--	2640	124
				960	17	2760	125
				1080	16	2880	156
				1200	18	3000	254
				1320	20	3120	257
				1440	28	3240	270
				1560	30	3360	319
				1680	31	3480	- -

SPECIMEN NO.3 (continued)

Run No.3 (cont.)		Run No.4		Run No.5		Run No.6	
F_N = 201.29 lb.		F_N = 168.16 lb.		F_N = 117.41 lb.		F_N = 64.78 lb.	
Q_A	ψ_{sR}	Q_A	ψ_{sR}	Q_A	ψ_{sR}	Q_A	ψ_{sR}
2520	97	120	--	120	--	120	--
2640	124	240	--	240	15	240	--
2760	125	360	--	360	16	360	5
2880	156	480	--	480	13	480	16
3000	254	600	--	600	24	600	19
3120	257	720	2	720	19	720	317
3240	270	840	5	840	28	840	--
3360	319	960	8	960	29		
3480	--	1080	7	1080	29		
		1200	7	1200	41		
		1320	30	1320	58		
		1440	30	1440	75		
		1560	28	1560	109		
		1680	41	1680	--		
		1800	58				
		1920	67				
		2040	78				
		2160	142				
		2280	214				
		2400	--				

SPECIMEN NO.4

Material: C-1018 Steel

Surface Finish: 2/o Paper

	Run No.1	Run No.2	Run No.3	Run No.4	Run No.5	Run No.6
	F_N = 81.75 lb.	F_N = 116.3 lb.	F_N = 167 lb.	F_N = 200.3 lb.	F_N = 200.3 lb.	F_N = 251.2 lb.
Q_A	ψ_s	ψ_s	ψ_s	ψ_s	ψ_s	ψ_s
120	54	34	39	46	32	33
240	115	83	72	71	68	75
300	- -	--	--	83	--	--
360		114	108	--	105	107
420		- -	- -	114	- -	- -
480			146	- -	145	149
600			192	182	180	186
720			231	220	218	226
840			279	- -	258	266
900			- -	273	- -	- -
960				- -	294	308
1020				320	- -	- -
1080				- -	336	349
1140				372	- -	- -
1200				- -	373	379
1320					413	416
1440					- -	- -

SPECIMEN NO.5

Material: C-1018 Steel

Surface Finish: 4/o paper

Run No.1		Run No.2		Run No.3		Run No.4	
F_N = 116.3 lb.		F_N = 167 lb.		F_N = 200.2 lb.		F_N = 251.2 lb.	
Q_A	ψ_s	Q_A	ψ_s	Q_A	ψ_s	Q_A	ψ_s
120	33	120	39	120	30	120	- -
150	--	150	--	150	--	150	43
240	73	240	82	240	74	240	--
300	--	300	--	300	--	300	91
360	108	360	126	360	117	360	--
420	--	420	--	420	--	420	--
450	--	450	--	450	--	450	141
480	153	480	167	480	151	480	--
600	188	600	201	600	191	600	190
720	228	720	232	720	230	720	--
750	--	750	--	750	--	750	241
840	318	840	280	840	269	840	--
900	--	900	--	900	--	900	291
960	--	960	321	960	310	960	--
		1020	--	1020	--	1020	--
		1050	--	1050	--	1050	332
		1080	385	1080	351	1080	--
		1140	--	1140	--	1140	--
				1200	390	1200	393
				1320	430	1320	--
				1350	--	1350	438
						1440	--
						1500	492
						1560	--

SPECIMEN NO.5 (continued)

	Run No.5	Run No.6	Run No.7
	F_N = 168.2 lb.	F_N = 117.41 lb.	F_N = 64.78 lb.
Q_A	ψ_{sR}	ψ_{sR}	ψ_{sR}
120	- -	- -	- -
150	- -	- -	- -
240	- -	- -	16
300	- -	- -	- -
360	- -	3	16
420	- -	- -	- -
450	- -	- -	- -
480	- -	4	36
600	- -	19	64
720	- -	16	67
750	- -	- -	- -
840	- -	32	153
900	- -	- -	- -
960	6	40	
1020	- -	- -	
1050	- -	- -	
1080	6	68	
1140	- -	- -	
1200	10	94	
1320	15	122	
1350	- -	- -	
1440	19	155	
1500	- -	- -	
1560	23		
1680	39		
1800	64		

SPECIMEN NO.6

Material: A-4140 Steel

Control Specimen (without interface)

	Run No.1	Run No.2
	F_N = 251.2 lb.	F_N = 200.2 lb.
Q_A	ψ_s	ψ_s
120	43	--
150	--	52
240	83	--
300	--	93
360	123	--
450	--	139
480	161	--
600	202	192
720	245	--
750	--	239
840	290	--
900	--	290
960	326	--
1020	--	
1080	363	
1200	400	

SPECIMEN NO.7

Material: A-4140 Steel

Surface Finish: As machined

	<u>Run No.1</u>	<u>Run No.2</u>	<u>Run No.3</u>	<u>Run No.4</u>
	F_N = 116.3 lb.	F_N = 167 lb.	F_N = 200.2 lb.	F_N = 251.2 lb.
Q_A	ψ_s	ψ_s	ψ_s	ψ_s
120	32	38	38	42
150	--	--	--	51
240	74	78	--	--
300	--	--	108	119
360	123	130	--	--
450	--	--	--	--
480	169	187	--	--
600	219	236	214	211
720	--	294	270	--
750	--	--	--	--
840	--	--	--	--
900	--	--	356	344
960	--	--	--	--
1020				397

SPECIMEN NO.8

Material: A-4140 Steel

Surface Finish: No.1 Paper

	Run No.1	Run No.2	Run No.3	Run No.4
	F_N = 116.3 lb.	F_N = 167 lb.	F_N = 200.2 lb.	F_N = 251.2 lb.
Q_A	ψ_s	ψ_s	ψ_s	ψ_s
150	54	59	48	51
300	111	116	109	109
450	165	168	178	161
600	241	223	220	219
750	284	277	276	287
900	--	--	339	340
1050			--	393

SPECIMEN NO.8 (continued)

	Run No.5	Run No.6		Run No.7
	F_N = 200 lb.	F_N = 167 lb.		F_N = 201.29 lb.
Q_A	ψ_s	ψ_s	Q_A	ψ_s
150	59	49	120	--
300	111	122	240	--
450	160	186	360	--
600	220	219	480	--
750	--	280	600	--
900		--	720	--
			840	--
			960	1
			1080	4
			1200	5
			1320	8
			1440	8
			1560	12
			1680	22
			1800	36
			1920	70
			2040	75
			2160	132
			2280	232
			2400	--

SPECIMEN NO.8 (continued)

	Run No.8	Run No.9	Run No.10
	F_N = 168.16 lb.	F_N = 117.41 lb.	F_N = 64.78 lb.
Q_A	ψ_{sR}	ψ_{sR}	ψ_{sR}
120	--	--	--
240	--	--	15
360	--	12	73
480	--	22	772
600	10	57	--
720	35	91	
840	42	159	
960	79	283	
1080	85	--	
1200	82		
1320	99		
1440	115		
1560	135		
1680	209		
1800	225		
1920	239		
2040	266		
2160	475		
2280	--		

SPECIMEN NO.10

Material: 2S Aluminum

Control Specimen: without interface

	Run No.1		Run No.2		Run No.3	
	F_N = 116 lb.		F_N = 16 lb.		F_N = 200 lb.	
Q_A	ψ_s	ψ_{sR}	ψ_s	ψ_s	ψ_s	ψ_s
30	28	--	22	6	--	--
60	60	--	50	5	53	7
90	96	19	--	--	--	--
120	119	19	100	4	101	1
150	152	30	--	--	--	--
180	181	26	135	21	--	--
210	211	32	--	--	--	--
240	250	39	221	30	229	29
270	271	40	--	--	--	--
300	296	42	272	31	274	29
330	323	35	--	--	--	--
360	360	44	336	35	337	46
420	--	--	393	53	--	--

SPECIMEN NO.11

Material: Copper

Control Specimen
(without interface)

	Run No.1		Run No.2		Run No.3	
	F_N = 20 lb.		F_N = 250 lb.		F_N = 200 lb.	
Q_A	ψ_s	ψ_{sR}	ψ_s	ψ_{sR}	Q_A	ψ_{sR}
30	29	--	24	4	120	--
60	48	4	--	--	240	5
90	68	14	61	7	360	5
120	85	4	--	--	480	8
150	92	6	100	4	600	10
180	117	14	--	--	720	19
210	138	28	137	--	840	28
240	154	22	--	--	960	27
270	173	28	170	15	1080	25
300	191	37	--	--	1200	40
330	211	35	205	18	1320	50
360	228	37	--	--	1440	58
390	248	44	236	22	1560	71
420	261	48	--	--	1680	87
450	281	56	--	--	1800	84

SPECIMEN NO.12

Material: A-4140 Steel

Surface Finish: 4/0 Paper

	Run No.1		Run No.2		Run No.3		Run No.4	
	F_N = 31.875 lb.		F_N = 53.38 lb.		F_N = 117.41 lb.		F_N = 168.2 lb.	
Q_A	ψ_s	ψ_{sR}	ψ_s	ψ_{sR}	ψ_s	ψ_{sR}	ψ_s	ψ_{sR}
120	54	--	59	10	45	--	--	--
150	--	--	--	--	53	--	58	--
240	94	4	116	16	100	--	--	--
300	--	--	--	--	122	--	105	--
360	156	13	176	28	142	1	--	--
450	--	--	--	--	180	--	164	--
480	277	72	265	75	186	4	--	--
600	--	--	365	225	228	16	231	14
720			562	269	265	58	--	--
750			--	--	270	--	283	10
840					295	60	--	--
960					345	76		
1080						92		
1200						100		
1320						149		
1440						183		
1560						267		
1680						659		

SPECIMEN NO.12 (continued)

	Run No.5		Run No.6		Run No.7	Run No.8	
	F_N = 168.2 lb.		F_N = 201.3 lb.		F_N = 201.3 lb.	F_N = 250 lb.	
Q_A	ψ_s	ψ_{sR}	ψ_s	ψ_{sR}	ψ_s	ψ_s	ψ_{sR}
150	54	--	60	--	59	69	--
300	113	--	119	--	113	123	--
450	162	--	178	--	174	174	--
600	225	8	237	3	236	238	--
750	273	14	302	17	303	294	12
900	387	172	371	23	350	351	12
1050	--	--	446	30	--	423	22

SPECIMEN NO.13

Material: Copper

Surface Finish: No.1 Paper

	Run No.1		Run No.2		Run No.3		Run No.4
	F_N = 168.2 lb.		F_N = 201.3 lb.		F_N = 251 lb.		F_N = 64.78 lb.
Q_A	ψ_s	ψ_{sR}	ψ_s	ψ_{sR}	ψ_s	ψ_{sR}	ψ_{sR}
60	45	--	26	--	--	--	--
120	75	7	68	--	54	--	--
180	116	9	111	13	--	--	--
240	151	10	151	6	95	--	1
300	--	--	190	13	--	--	--
360			233	12	220	10	49
420			255	27	--	--	--
480			295	24	292	4	
540			335	25	--	--	
600			380	32	378	16	
660			415	40	--	--	
720			455	38	463	38	
780			498	38	--	--	
840			542	39	546	54	

SPECIMEN NO.13 (continued)

	Run No.4	Run No.5	Run No.6	Run No.7		Run No.7
	F_N = 64.78 lb.	F_N = 117.4 lb.	F_N = 168.16 lb.	F_N = 201.29 lb.		F_N = 201.29 lb.
	ψ_{sR}	ψ_{sR}	ψ_{sR}	ψ_{sR}		ψ_{sR}
120	1	--	--	--	120	--
240	--	2	8	--	240	1
360	49	11	7	8	360	8
480	--	13	5	15	480	15
600		16	8	13	600	8
720		26	17	19	720	19
840		--	42	28	840	28
960			63	40	960	40
1080			81	44	1080	43
1200			--	48	1200	40
1320				72	1320	72
1440				184	1440	185
1560				246	1560	246
1680				--	1680	--

SPECIMEN NO.14

Material: Copper

Surface Finish: 2/0 Paper

	Run No.1		Run No.2		Run No.3	
	F_N = 64.8 lb.		F_N = 117.4 lb.		F_N = 168.2 lb.	
Q_A	ψ_s	ψ_{sR}	ψ_s	ψ_{sR}	ψ_s	ψ_{sR}
120	73	--	66	--	64	--
240	152	16	136	5	133	--
360	237	26	219	16	205	--
480	341	44	304	33	269	--
600	464	289	393	57	353	5
720	--	--	501	90	431	13
840			521	124	501	15
960			--	294	586	21
1080				873	--	31
1200				--		34
1320						37
1440						64
						--

SPECIMEN NO.14 (continued)

Run No.4

F_N = 201.3 lb.

Q_A	ψ_s	ψ_{sR}
120	68	--
240	148	16
360	224	23
480	303	38
600	374	48
720	468	63
840	550	67
960	630	79
1080	--	88
1200		95
1320		119
1440		135
1560		178
1680		194
1800		218
1920		258
2040		--

Run No.5

F_N = 231.68 lb.

Q_A	ψ_s	ψ_{sR}
120	77	--
240	113	14
360	228	28
480	309	37
600	383	43
720	465	54
840	543	66
960	633	85
1080	--	84
1200		96
1320		117
1440		118
1560		139
1680		155
1800		165
1920		180
2040		203
2160		235
2280		261
2400		292
2520		--

SPECIMEN NO.15

Material: Copper

Surface Finish: 4/0 Paper

	Run No.1		Run No.2	Run No.3	Run No.4	Run No.5
	F_N = 201.3 lb.		F_N = 168.2 lb.	F_N = 117.4 lb.	F_N = 168.2 lb.	F_N = 117.4 lb.
C_A	ψ_s	ψ_{sR}	ψ_{sR}	ψ_{sR}	ψ_{sR}	ψ_{sR}
120	70	--	--	--	--	7
240	143	--	1	--	9	28
360	207	6	2	--	13	52
480	288	--	--	2	17	122
600	365	14	1	11	30	--
720	--	1	3	13	36	
840		5	3	20	67	
960		3	6	--	99	
1080		6	9		--	
1200		11	17			
1320		22	19			
1440		31	20			
1560		40	27			
1680		57	54			
1800		64	--			
1920		--				

SPECIMEN NO.15 (continued)

Material: Copper

Surface Finish: 4/0 Paper

	Run No.6	Run No.7
	F_N = 64.8 lb.	F_N = 64.8 lb.
Q_A	ψ_{sR}	ψ_{sR}
120	-9	8
240	2	21
360	33	82
480	--	741
600		--

SPECIMEN NO.16

Material: 2S Aluminum

Surface Finish: No.1 Paper

	Run No.1		Run No.2	
	F_N = 168.2 lb.		F_N = 201.3 lb.	
Q_A	ψ_s	ψ_{sR}	ψ_s	ψ_{sR}
12	16	4	--	--
30	36	12	33	--
60	73	12	52	--
90	93	24	90	10
120	126	28	112	12
150	163	37	143	16
180	182	52	172	12
210	215	61	194	18
240	246	55	229	22
270	273	63	260	16
300	301	66	283	32
330	343	71	--	--
360	368	73		
390	395	83		
420	420	86		

DEFINITION OF SYMBOLS

A = area (square inches)

D_i = inside diameter of specimen (inches)

D_o = outside diameter of specimen (inches)

E = modulus of elasticity (Youngs modulus) (lb./in^2)

E_s = G = modulus of rigidity (shearing modulus) (lb/in^2)

F_N = force normal to interface (normal load on specimen) (pounds or kilograms)

L = gage length or length along specimen between indicator arms (inches)

Q_A = applied torque (gram inches)

R = radius of indicator arm (inches)

r_m = mean radius of specimen (inches)

S_N = normal stress at interface (lb/in^2)

s_n = normal stress at interface (kg/cm^2)

S_p = principal stress in specimen (lb/in^2)

S_s = combined shear stress in specimen (lb/in^2)

S_T = maximum tangential stress in interface due to applied torque (lb/in^2)

s_t = maximum tangential stress in interface (kg/cm^2)

T_A = torque applied to specimen (lb. inches)

ϕ_c = calculated angle of twist for specimen without interface (minutes of arc)

ϕ_o = observed angle of twist in specimen (minutes of arc)

ϕ_R = residual angle of twist in specimen (minutes of arc)

ϕ_s = observed angle of twist in specimen (micrometer drum units)

ϕ_{sR} = residual angle of twist in specimen (micrometer drum units)

θ_R = apparent slip at interface (centimeters)

DEFINITION OF SYMBOLS

A = area (square inches)

D_i = inside diameter of specimen (inches)

D_o = outside diameter of specimen (inches)

E = modulus of elasticity (Young's modulus) (lb./in^2)

E_s = G = modulus of rigidity (shearing modulus) (lb/in^2)

F_N = force normal to interface (normal load on specimen) (pounds or kilograms)

L = gage length or length along specimen between indicating arms (inches)

M_t = applied torque (gram inches)

R = radius of indicator arm (inches)

r_m = mean radius of specimen (inches)

S_N = normal stress at interface (lb/in^2)

s_n = normal stress at interface (kg/cm^2)

S_P = principal stress in specimen (lb/in^2)

S_s = combined shear stress in specimen (lb/in^2)

S_T = maximum tangential stress in interface due to applied torque (lb/in^2)

s_t = maximum tangential stress in interface (kg/cm^2)

T_A = torque applied to specimen (lb. inches)

θ_s = calculated angle of twist for specimen without interface (minutes of arc)

θ_m = observed angle of twist in specimen (minutes of arc)

θ_r = residual angle of twist in specimen (minutes of arc)

θ_o = observed angle of twist in specimen (micrometer drum units)

θ_{ro} = residual angle of twist in specimen (micrometer drum units)

Δ_s = apparent slip at interface (centimeters)

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